CHAPTER 1 - BASIC CONCEPTS

List of topics for this chapter :

- Systems and Units
- Charge and Current
- Voltage, Power, and Energy
- Circuit Elements

SYSTEMS AND UNITS

Problem 1.1 Simplify the following units. Try to express each as a single unit.

(a) $\dfrac{\text{volt} \cdot \text{coulomb}}{\text{second}}$ (b) $\text{volt} \cdot \text{ampere} \cdot \text{second}$ (c) $\dfrac{\text{second}}{\text{volt} \cdot \text{coulomb}}$

(a) **watt** (b) **joule** (c) **1 / watt**

CHARGE AND CURRENT

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

Electric current is the time rate of change of charge, measured in amperes (A). A direct current (dc) is a current that remains constant with time. An alternating current (ac) is a current that varies sinusoidally with time.

Problem 1.2 The charge transferred in time is given in Figure 1.1. Determine the current, $i(t)$, flowing through the wire.

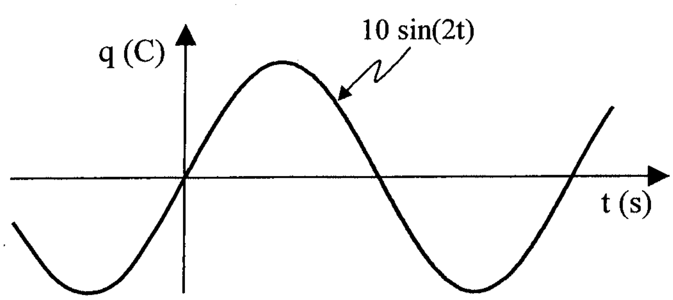


Figure 1.1

The relationship between current, charge, and time is

$$i(t) = dq(t)/dt$$

Therefore,

$$i(t) = \frac{d}{dt}(10\sin(2t))$$
$$i(t) = \underline{\mathbf{20\cos(2t)\ amps}}$$

Problem 1.3 *[1.5]** Determine the total charge flowing through an element for $0 < t < 2$ seconds when the current entering its positive terminal is $i(t) = e^{-2t}$ mA.

$$q(t) = \int i(t)\,dt$$

$$q = \int_0^2 e^{-2t}\,dt = \frac{-1}{2}e^{-2t}\Big|_0^2 = \left(\frac{-1}{2}e^{-4} - \frac{-1}{2}e^{0}\right) = \frac{-1}{2}\left(e^{-4} - 1\right)$$

$$q = \underline{\mathbf{0.4908\ mC}}$$

Problem 1.4 The current flowing through a wire is given by Figure 1.2. Determine the net charge moving through the wire if $q(0) = 0$.

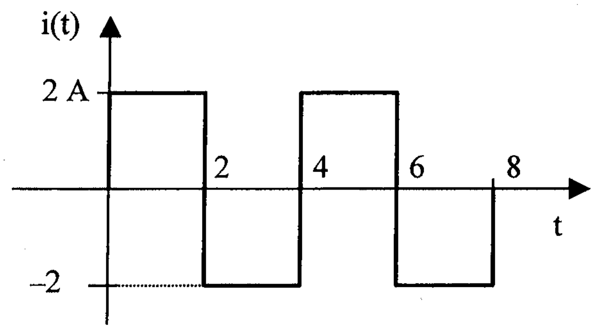
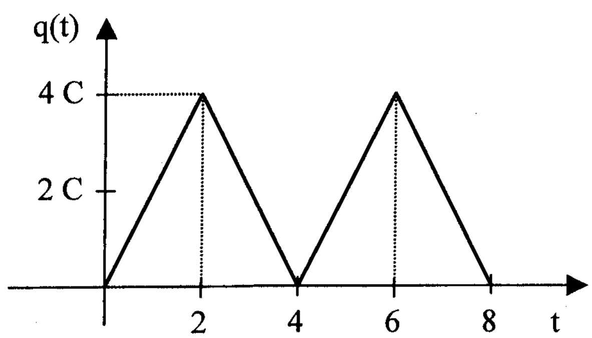


Figure 1.2

The net charge moving through the wire is



$$q(t) = \begin{cases} \mathbf{2t\ C} & \mathbf{0 < t < 2} \\ \mathbf{-2t + 8\ C} & \mathbf{2 < t < 4} \\ \mathbf{2t - 8\ C} & \mathbf{4 < t < 6} \\ \mathbf{-2t + 16\ C} & \mathbf{6 < t < 8} \end{cases}$$

* This indicates a problem from Fundamentals of Electric Circuits by Alexander and Sadiku.

VOLTAGE, POWER, AND ENERGY

Voltage (or potential difference) is the energy required to move a unit charge through an element, measured in volts (V). Power is the time rate of expending or absorbing energy, measured in watts (W). Energy is the capacity to do work, measured in joules (J).

Passive sign convention is satisfied when the current enters through the positive terminal of an element and $p = +vi$. If the current enters through the negative terminal, $p = -vi$.

Problem 1.5 Given $v = 10$ volts and $i(t)$ as shown in Figure 1.3, sketch the power and energy.

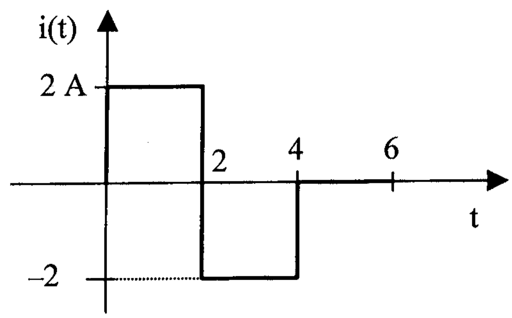


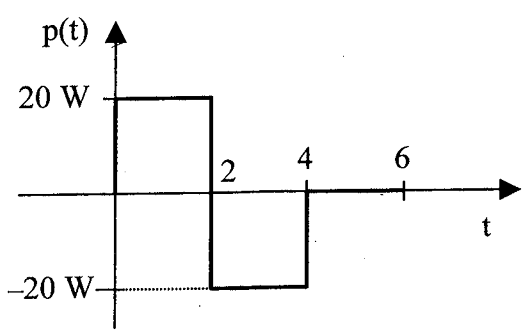
Figure 1.3

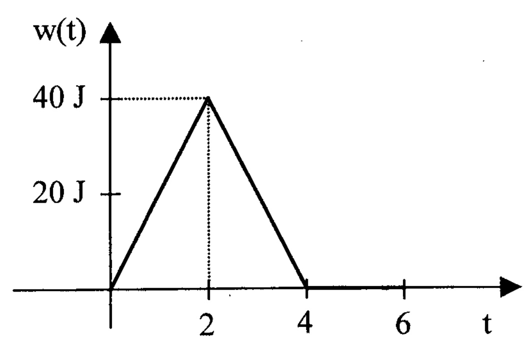
$$i(t) = \begin{cases} 2\text{ A} & 0 < t < 2 \\ -2\text{ A} & 2 < t < 4 \\ 0\text{ A} & 4 < t < 6 \end{cases}$$

Therefore,

$$p(t) = v(t)\,i(t) = \begin{cases} \mathbf{20\ W} & \mathbf{0 < t < 2} \\ \mathbf{-20\ W} & \mathbf{2 < t < 4} \\ \mathbf{0\ W} & \mathbf{4 < t < 6} \end{cases} \qquad w(t) = \int p(t)\,dt = \begin{cases} \mathbf{20t\ J} & \mathbf{0 < t < 2} \\ \mathbf{80 - 20t\ J} & \mathbf{2 < t < 4} \\ \mathbf{0\ J} & \mathbf{4 < t < 6} \end{cases}$$

Sketches of the power and energy are shown in the figures below.





Problem 1.6 *[1.13]* Figure 1.4 shows the current through and the voltage across a device. Find the total energy absorbed by the device for the period $0 < t < 4$ seconds.

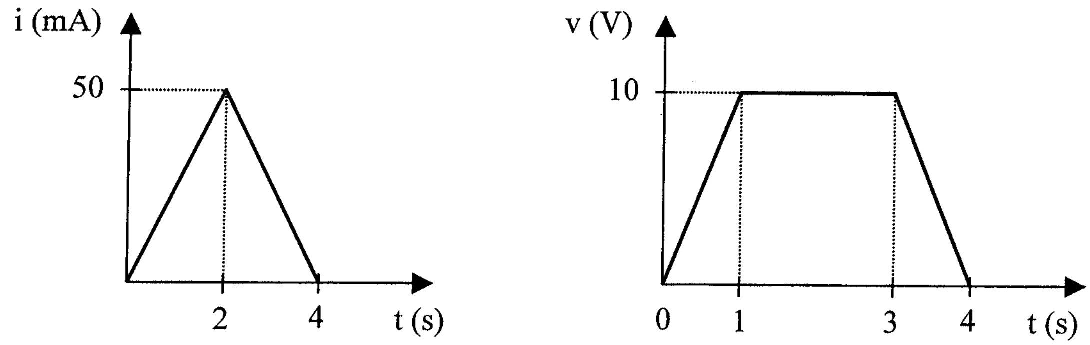


Figure 1.4

The curves in Figure 1.4 are represented by

$$i(t) = \begin{cases} 25t \text{ mA} & 0 < t < 2 \\ 100 - 25t \text{ mA} & 2 < t < 4 \end{cases} \qquad v(t) = \begin{cases} 10t \text{ V} & 0 < t < 1 \\ 10 \text{ V} & 1 < t < 3 \\ 40 - 10t \text{ V} & 3 < t < 4 \end{cases}$$

So,

$$w(t) = \int v(t)\, i(t)\, dt$$

$$w = \int_0^1 (10t)(25t)\, dt + \int_1^2 (10)(25t)\, dt + \int_2^3 (10)(100 - 25t)\, dt + \int_3^4 (40 - 10t)(100 - 25t)\, dt$$

$$w = 250\left[\int_0^1 t^2\, dt + \int_1^2 t\, dt + \int_2^3 (4 - t)\, dt + \int_3^4 (16 - 8t + t^2)\, dt\right]$$

$$w = 250\left[\frac{t^3}{3}\Big|_0^1 + \frac{t^2}{2}\Big|_1^2 + \left(4t - \frac{t^2}{2}\right)\Big|_2^3 + \left(16t - 4t^2 + \frac{t^3}{3}\right)\Big|_3^4\right]$$

$$w = 250\left[\left(\frac{1}{3} - 0\right) + \left(\frac{4}{2} - \frac{1}{2}\right) + \left(12 - \frac{9}{2} - 8 + \frac{4}{2}\right) + \left(64 - 64 + \frac{64}{3} - 48 + 36 - \frac{27}{3}\right)\right]$$

$$w = 250\,[0.3333 + 1.5 + 1.5 + 0.3333]$$

$$w = \underline{\mathbf{916.7\ mJ}}$$

Problem 1.7 Given that the power absorbed by an element is $p(t) = 10\cos^2(4t)$ watts and the current through the element is $i(t) = 20\cos(4t)$ mA, find the voltage across the element.

$$v(t) = \underline{\mathbf{500\cos(4t)\ volts}}$$

CIRCUIT ELEMENTS

An electric circuit is an interconnection of electrical elements.

An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit variables. An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.

Problem 1.8 Given the circuit in Figure 1.5, find V_o.

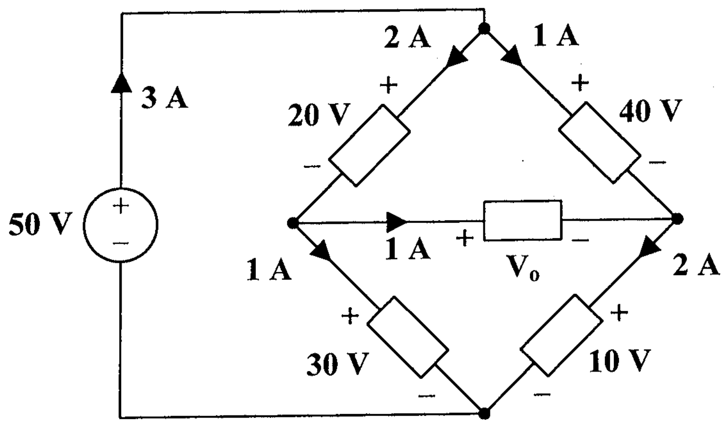


Figure 1.5

This circuit, like any other electric circuit, must obey the law of conservation of energy. Hence,

$$\sum p = 0$$

The power absorbed by the voltage source is

$$p_{abs_{50V}} = \text{-}(50)(3) = \text{-}150 \text{ watts}$$

The power absorbed by each circuit element in the bridge is

$$p_{abs_{20V}} = (20)(2) = 40 \text{ watts} \qquad p_{abs_{40V}} = (40)(1) = 40 \text{ watts}$$

$$p_{abs_{V_o}} = (V_o)(1) = V_o$$

$$p_{abs_{30V}} = (30)(1) = 30 \text{ watts} \qquad p_{abs_{10V}} = (10)(2) = 20 \text{ watts}$$

Thus,

$$\sum p = 0 = \text{-}150 + 40 + 40 + p_{abs_{V_o}} + 30 + 20$$

$$p_{abs_{V_o}} = 150 - 40 - 40 - 30 - 20$$

$$p_{abs_{V_o}} = 20 \text{ watts}$$

but

$$p_{abs_{V_o}} = (V_o)(1) = V_o$$

Therefore,

$$V_o = \underline{\textbf{20 volts}}$$

Problem 1.9 *[1.17]* Find V_o in the circuit of Figure 1.6.

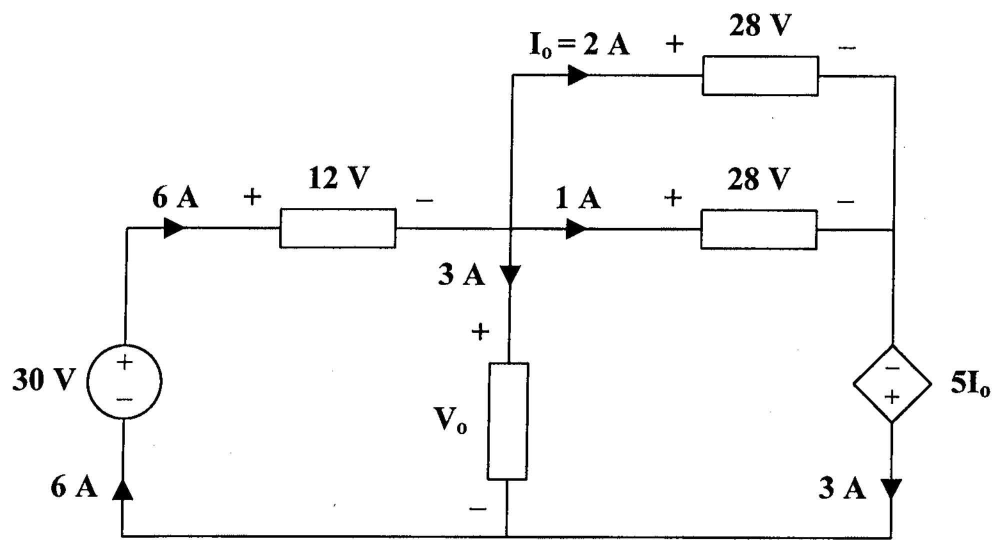


Figure 1.6

Since $\sum p = 0$,

$$-(30)(6) + (12)(6) + 3V_o + (28)(I_o) + (28)(1) - (5I_o)(3) = 0$$

where $I_o = 2$.

$$-180 + 72 + 3V_o + (28)(2) + 28 - (5)(2)(3) = 0$$
$$3V_o + 156 = 210$$
$$3V_o = 54$$
$$V_o = \underline{\textbf{18 volts}}$$

CHAPTER 2 - BASIC LAWS

List of topics for this chapter :

- Ohm's Law
- Nodes, Branches, and Loops
- Kirchoff's Laws
- Series and Parallel Resistors
- Wye-Delta Transformations
- Applications

OHM'S LAW

Ohm's law states that the voltage across a resistor is directly proportional to the current flowing through the resistor.

$$v = i R$$

Problem 2.1 Given an incandescent light bulb rated at 75 watts and 120 volts, find the "hot" resistance and "cold" resistance of the light bulb.

An incandescent light bulb is the most common source of light produced by electrical energy. A current is made to flow through a wire, called a filament. Since this wire has resistance, the wire absorbs power, so much so that it "glows" and gives off light. Ratings on these types of light bulbs are for steady-state operating conditions. So, we have

$$I = \frac{P}{V} = \frac{75}{120} = \frac{25}{40} \text{ amps}$$

$$R = \frac{V}{I} = \frac{120}{25/40} = 192 \text{ ohms}$$

Hence, **<u>the "hot" resistance is 192 ohms</u>**.

Most wire has what we call a positive thermal resistance characteristic; that is, as the wire temperature increases, the resistance increases.

It is sufficient to say at this time that **<u>the "cold" resistance is less than 192 ohms</u>**.

Problem 2.2 Solve for v, i, and R for the following :

(a) $R = 10$ ohms and $i = 2$ amps
(b) $R = 5$ ohms and $v = 20$ volts
(c) $v = 10$ volts and $i = 5$ amps
(d) $i = 3$ amps and $v = 20$ volts

(a) v = **20 volts**

(b) i = **4 amps**

(c) R = **2 ohms**

(d) R = **6.667 ohms**

NODES, BRANCHES, AND LOOPS

A branch represents a single element, such as a voltage source or a resistor. A node is the point of connection between two or more branches. A loop is any closed path in a circuit.

Problem 2.3 *[2.7]* Determine the number of branches and nodes in the circuit in Figure 2.1.

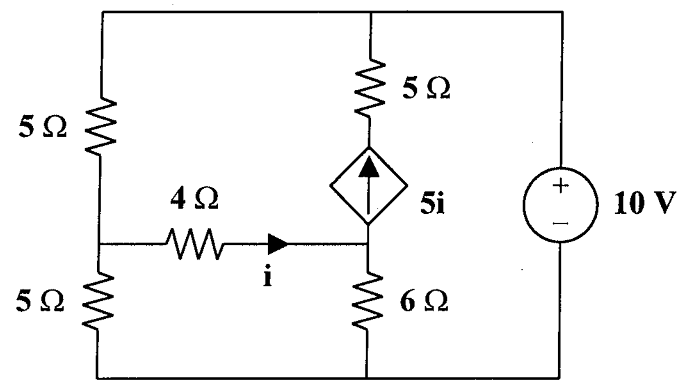
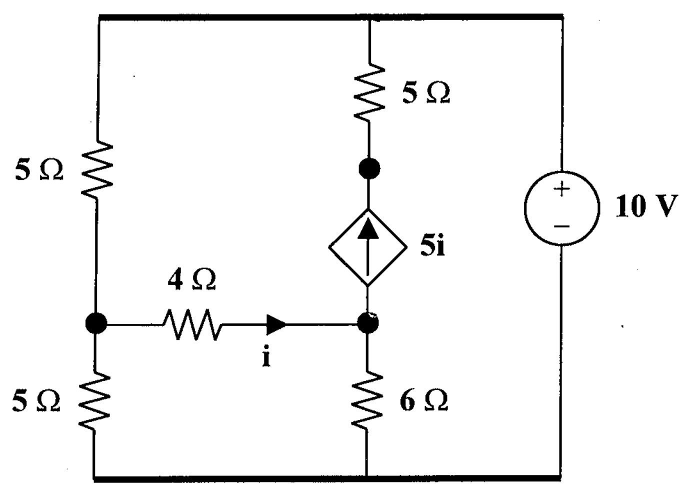


Figure 2.1

There are 7 elements, 1 dependent current source, 1 independent voltage source, and 5 resistors, which implies that there are **7 branches**. There are **5 nodes** as indicated by the dark circles and dark lines in the circuit below.



Problem 2.4 Identify all the nodes, branches, and independent loops in Figure 2.2.

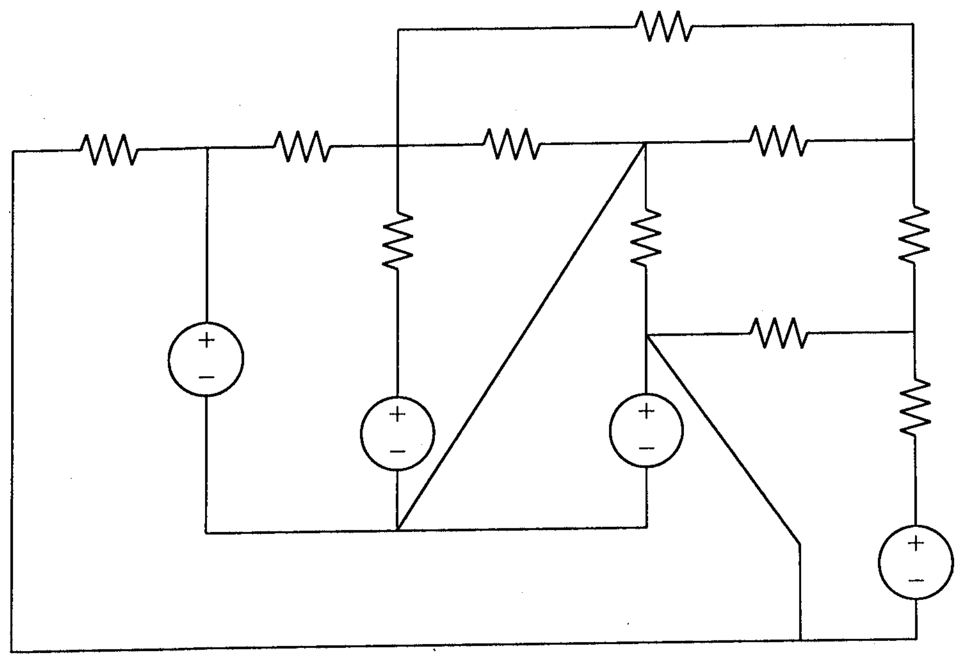
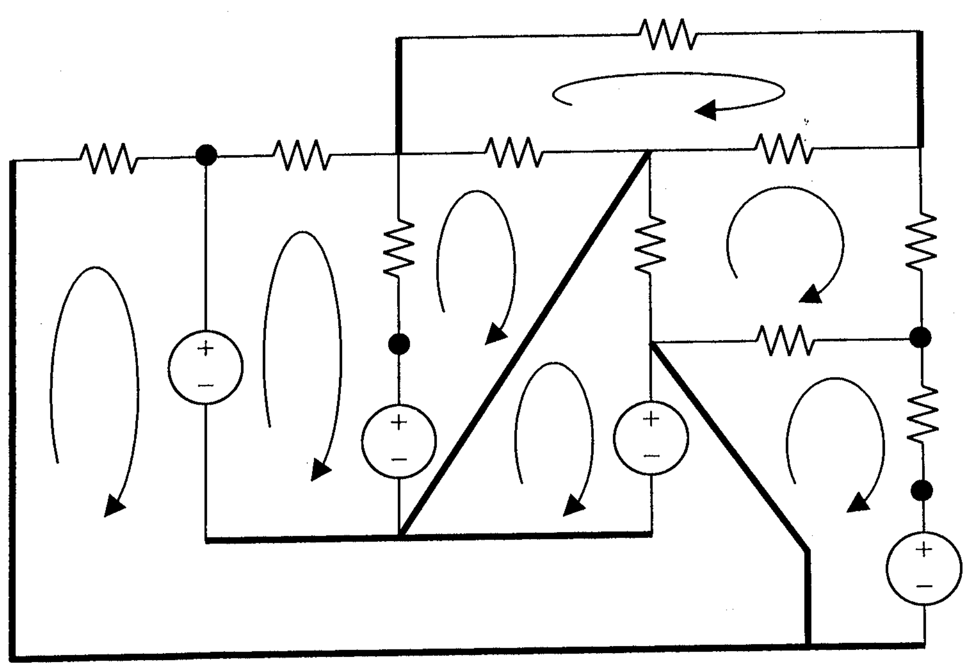


Figure 2.2

There are **<u>8 nodes</u>**, as indicated by the dark circles and dark lines in the circuit below. There are **<u>14 branches</u>**, 4 independent voltage sources and 10 resistors. There are **<u>7 independent loops</u>**.



Check. Does this satisfy the fundamental theorem of network topology?

$$b = l + n - 1 = 7 + 8 - 1 = 14 \qquad \text{YES!}$$

Problem 2.5 In **Figure 2.2**, is there a loop? How many nodes are there?

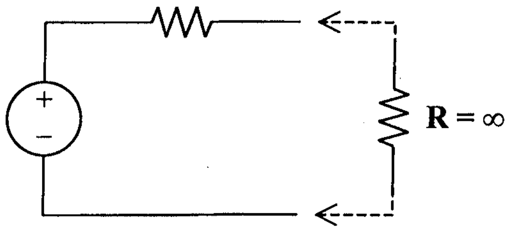


Figure 2.2

A loop is any *closed* path; therefore it is easy to say there is no loop. However, there really is a loop since there is an infinite resistance connecting the end terminals together. Thus, **there is one loop**. There are **three nodes**, one where the voltage source and resistor join and the two at the output terminals. In addition, there are **three branches**. The voltage source, the resistor, and the infinite resistance. Thus, the fundamental theorem of network topology is satisfied.

Problem 2.6

(a) In a circuit containing 26 branches and 12 nodes, how many independent loops will satisfy the fundamental theorem of network topology?

(b) In a circuit with 22 branches, is it possible to have 28 nodes?

$$b = l + n - 1 \qquad \textit{\{the fundamental theorem of network topology\}}$$

(a) $26 = l + 12 - 1$
$l = 26 - 12 + 1$
$l = 15$
A circuit with 26 branches and 12 nodes will have **15 independent loops**.

(b) $22 = l + 28 - 1$
$l = 22 - 28 + 1$
$l = -5$
No. It is not possible to have a circuit with 22 branches and 28 nodes because a circuit cannot have –5 loops.

KIRCHOFF'S LAWS

Kirchoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero. Equivalently,

the sum of the currents entering a node = the sum of the currents leaving the node

Kirchoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero. Equivalently,

the sum of the voltage drops around a loop = the sum of the voltage rises around the loop

Problem 2.7 Determine all the currents and voltages in Figure 2.3.

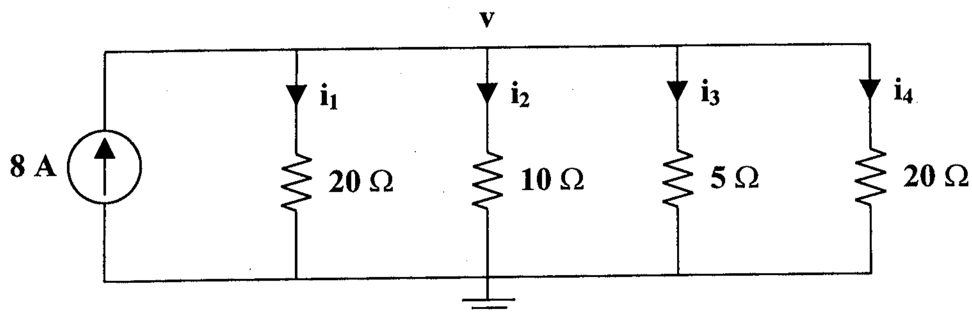


Figure 2.3

- **Carefully DEFINE the problem.**
  Each component is labeled completely. The problem is clear.

- **PRESENT everything you know about the problem.**
  The goal of the problem is to determine v, i_1, i_2, i_3, and i_4.

- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
  KCL, involving currents entering and leaving a node, is the obvious circuit analysis technique to use, especially since the circuit has only one node. KVL could be used but would be more complicated due to the fact that there are five loops in the circuit. Using KVL creates four equations with four unknowns (since the fifth loop current is the current source), compared to one equation and four unknowns using KCL.

  With one equation and four unknowns, constraint equations must be found. Realizing that v is the voltage across each component is the key. Ohm's law will be used to determine the relationship between the voltage and each of the four unknown currents. Using substitution, an equation in terms of v can be found and solved.

- **ATTEMPT a problem solution.**

KCL : $8 = i_1 + i_2 + i_3 + i_4$

Ohm's law : $v = 20i_1 = 10i_2 = 5i_3 = 20i_4$

$i_1 = v/20 \qquad i_2 = v/10 \qquad i_3 = v/5 \qquad i_4 = v/20$

Substitute the current equations found using Ohm's law into the equation found using KCL. Then, solve for v.

$$8 = v/20 + v/10 + v/5 + v/20$$
$$160 = v + 2v + 4v + v$$
$$160 = 8v$$
$$v = 20 \text{ volts}$$

Hence,

$$i_1 = v/20 = 20/20 = 1 \text{ amp} \qquad i_2 = v/10 = 20/10 = 2 \text{ amps}$$
$$i_3 = v/5 = 20/5 = 4 \text{ amps} \qquad i_4 = v/20 = 20/20 = 1 \text{ amp}$$

- **EVALUATE the solution and check for accuracy.**
  Check the node equation found using KCL,

$$8 = i_1 + i_2 + i_3 + i_4$$
$$8 = 1 + 2 + 4 + 1$$
$$8 = 8$$

  This is a valid equation. Thus, our check for accuracy was successful.

- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
  This problem has been solved satisfactorily.

$v = \underline{\textbf{20 volts}}$

$i_1 = \underline{\textbf{1 amp}}$ $\qquad$ $i_2 = \underline{\textbf{2 amps}}$ $\qquad$ $i_3 = \underline{\textbf{4 amps}}$ $\qquad$ $i_4 = \underline{\textbf{1 amp}}$

Problem 2.8 *[2.15]* Find I and V_{ab} in the circuit of Figure 2.4.

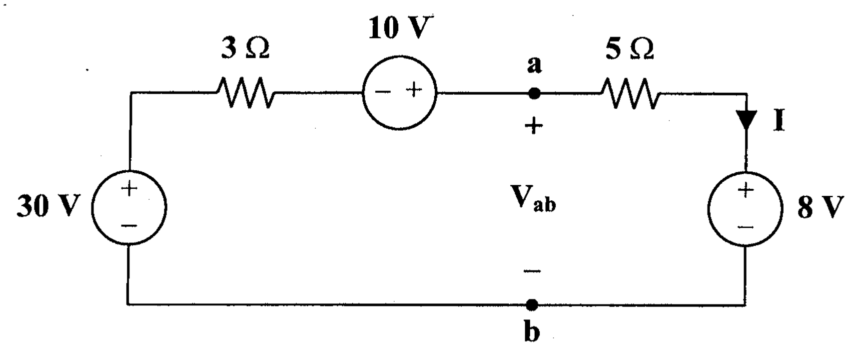


Figure 2.4

Applying KVL to the loop,

$$-30 + 3I - 10 + 5I + 8 = 0$$
$$8I = 32$$
$$I = \underline{\textbf{4 amps}}$$

So,

$$-V_{ab} + 5I + 8 = 0$$
$$V_{ab} = 5I + 8 = (5)(4) + 8$$

Therefore,

$$V_{ab} = \underline{\textbf{28 volts}}$$

Problem 2.9 In Figure 2.5, solve for i_1 and i_2.

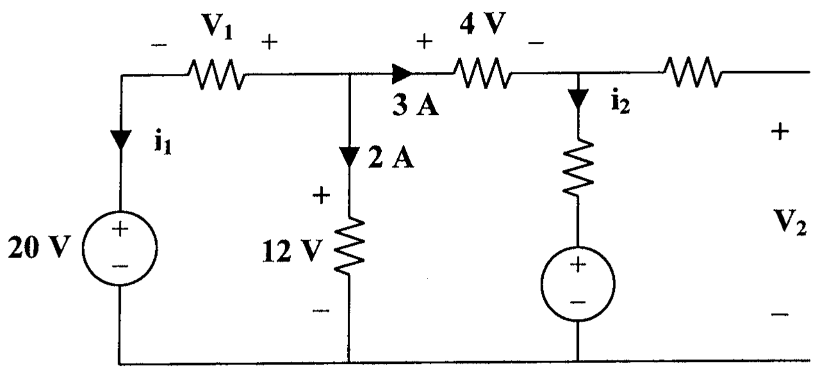
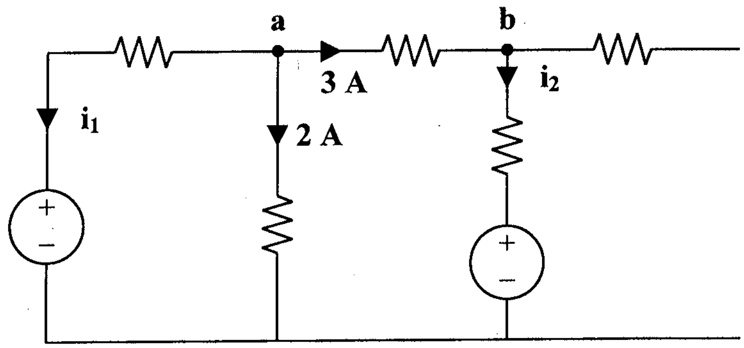


Figure 2.5



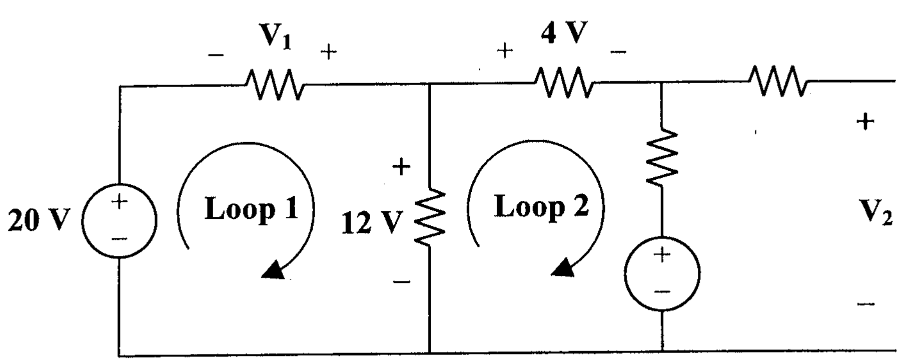
KCL (node a) :

$$i_1 + 3 + 2 = 0$$
$$i_1 = \underline{\mathbf{-5\ amps}}$$

KCL (node b) :

$$i_2 + 0 = 3$$
$$i_2 = \underline{\mathbf{3\ amps}}$$

Problem 2.10 In Figure 2.5, find V_1 and V_2.



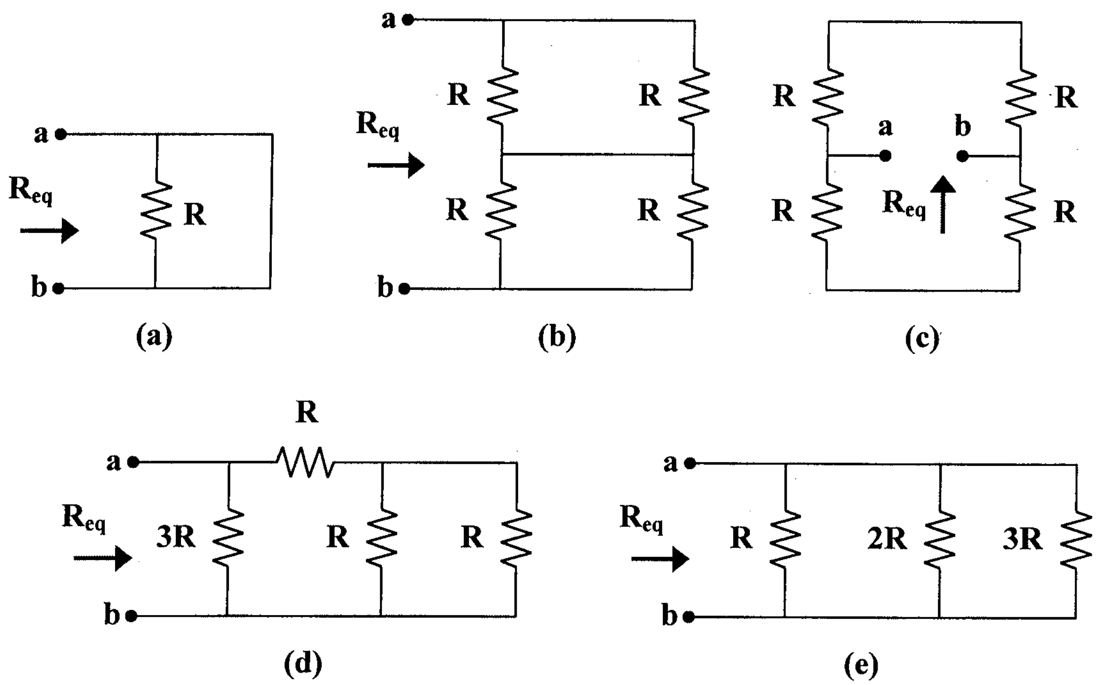
$V_1 = \underline{\mathbf{-8\ volts}}$

$V_2 = \underline{\mathbf{8\ volts}}$

SERIES AND PARALLEL RESISTORS

Two or more elements are in series if they are cascaded or connected sequentially and consequently carry the same current. Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

Problem 2.11 *[2.35]* Find the equivalent resistance at terminals a-b for each of the following networks.



(a) $R_{eq} = R \| 0 = 0$

(b) $R_{eq} = R \| R + R \| R = \frac{R}{2} + \frac{R}{2} = R$

(c) $R_{eq} = (R + R) \| (R + R) = 2R \| 2R = R$

(d) $R_{eq} = 3R \| (R + R \| R) = 3R \left\| \left(R + \frac{R}{2} \right) \right. = 3R \left\| \frac{3}{2}R \right. = \dfrac{(3R)\left(\frac{3}{2}R\right)}{3R + \frac{3}{2}R} = \dfrac{9R^2}{9R} = R$

(e) $R_{eq} = R \| 2R \| 3R = \left(\dfrac{(R)(2R)}{R + 2R} \right) \| 3R = \frac{2}{3}R \| 3R = \dfrac{\left(\frac{2}{3}R\right)(3R)}{\frac{2}{3}R + 3R} = \dfrac{6R^2}{11R} = \dfrac{6}{11}R$

Therefore,

(a) $R_{eq} = \underline{\mathbf{0}}$ (b), (c), (d) $R_{eq} = \underline{\mathbf{R}}$ and (e) $R_{eq} = \underline{\dfrac{\mathbf{6}}{\mathbf{11}}\mathbf{R}}$

Problem 2.12 Given N resistors in parallel, determine if they can be replaced by a single resistor.

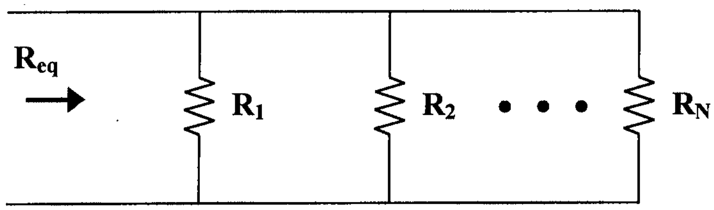
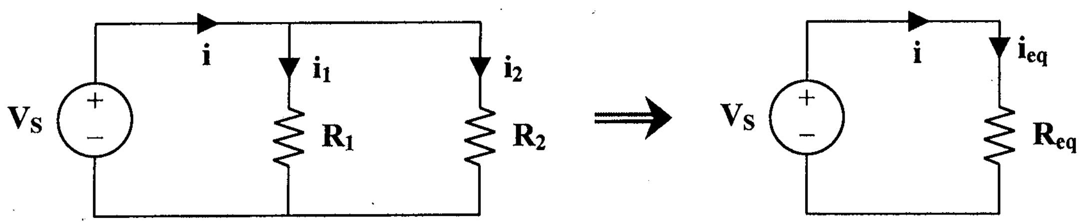


Figure 2.6

To find the equivalent resistance of the parallel resistors in Figure 2.6 combine the resistors two at a time to discover a pattern. First, consider the circuits below,



Note that two resistors in parallel have the same voltage across them. Ohm's law gives us

$$V_S = i_1 R_1 = i_2 R_2 \qquad\qquad V_S = i_{eq} R_{eq}$$

or

$$i_1 = \frac{V_S}{R_1} \quad \text{and} \quad i_2 = \frac{V_S}{R_2} \qquad\qquad i_{eq} = \frac{V_S}{R_{eq}}$$

Using KCL,

$$i = i_1 + i_2 \qquad\qquad i = i_{eq}$$

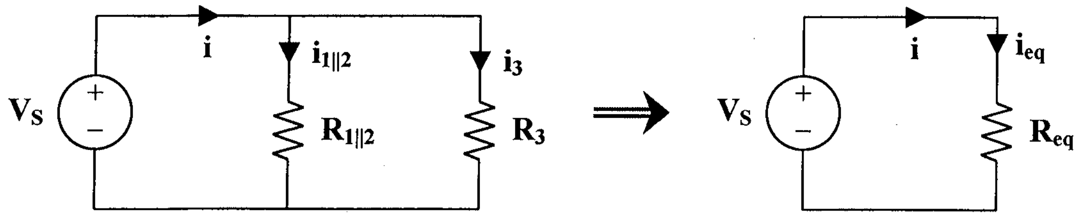
Use the equations for i_1 and i_2 and i_{eq} to find i in terms of the source voltage and resistors.

$$i = \frac{V_S}{R_1} + \frac{V_S}{R_2} = V_S\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad\qquad i = \frac{V_S}{R_{eq}}$$

Thus, $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{or} \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

This implies that the equivalent resistance of two resistors in parallel is the product of their resistances divided by the sum of their resistances.

Continue the process. Find the equivalent resistance of $R_{1\|2} = \dfrac{R_1 R_2}{R_1 + R_2}$ in parallel with R_3.



Note that two resistors in parallel have the same voltage across them. Ohm's law gives us

$$V_S = i_{1\|2} R_{1\|2} = i_3 R_3 \qquad\qquad V_S = i_{eq} R_{eq}$$

$$i_{1\|2} = \frac{V_S}{R_{1\|2}} \quad \text{and} \quad i_3 = \frac{V_S}{R_3} \qquad\qquad i_{eq} = \frac{V_S}{R_{eq}}$$

Using KCL,

$$i = i_{1\|2} + i_3 \qquad\qquad i = i_{eq}$$

Use the equations for $i_{1\|2}$ and i_3 and i_{eq} to find i in terms of the source voltage and resistors.

$$i = \frac{V_S}{R_{1\|2}} + \frac{V_S}{R_3} \qquad\qquad i = \frac{V_S}{R_{eq}}$$

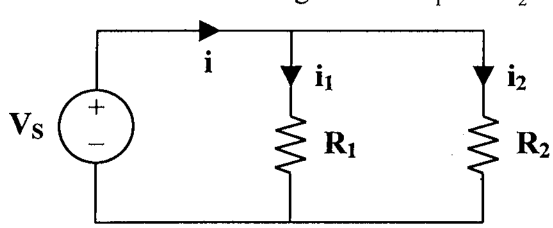
$$i = V_S\left(\frac{1}{R_{1\|2}} + \frac{1}{R_3}\right) = V_S\left(\frac{R_1 + R_2}{R_1 R_2} + \frac{1}{R_3}\right) = V_S\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

Thus, $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Continuing this process for N resistors would show that the reciprocal of the equivalent resistance of N resistors in parallel is the sum of the reciprocals of each resistance. In general,

$$\frac{1}{R_{eq}} = \underline{\frac{1}{\mathbf{R_1}} + \frac{1}{\mathbf{R_2}} + \cdots + \frac{1}{\mathbf{R_N}} = \sum_{n=1}^{N} \frac{1}{\mathbf{R_n}}}$$

Problem 2.13 If you know the current through two resistors in parallel (R_1 and R_2), is there a simple way to determine the current through either R_1 or R_2?



Clearly, the two resistors in parallel have the same voltage across them. Using Ohm's law,

$$V_S = i_1 R_1 = i_2 R_2$$

$$i_1 = \frac{V_S}{R_1} \quad \text{and} \quad i_2 = \frac{V_S}{R_2}$$

Using KCL,

$$i = i_1 + i_2$$

Now use the equations for i_1 and i_2 to find i in terms of the source voltage and resistors.

$$i = \frac{V_S}{R_1} + \frac{V_S}{R_2} = V_S\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = V_S\left(\frac{R_1 + R_2}{R_1 R_2}\right)$$

Then,

$$V_S = \left(\frac{R_1 R_2}{R_1 + R_2}\right) i$$

To find the branch currents, substitute the equation for V_S into the equations for i_1 and i_2.

$$i_1 = \frac{1}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right) i \qquad i_2 = \frac{1}{R_2}\left(\frac{R_1 R_2}{R_1 + R_2}\right) i$$

$$\underline{i_1 = \left(\frac{\mathbf{R_2}}{\mathbf{R_1 + R_2}}\right) \mathbf{i}} \qquad \underline{i_2 = \left(\frac{\mathbf{R_1}}{\mathbf{R_1 + R_2}}\right) \mathbf{i}}$$

Thus, it is clear that the current entering the node where two resistors are connected in parallel divides proportionately between the two resistors. The proportionality is equal to the value of the opposite resistor divided by the sum of the resistances times the incoming current. It should be noted that this current division property only works for two resistors in parallel. If you have more than two, you need to use a different process to find how the currents divide.

Problem 2.14 Determine R_{eq} for Figure 2.7.

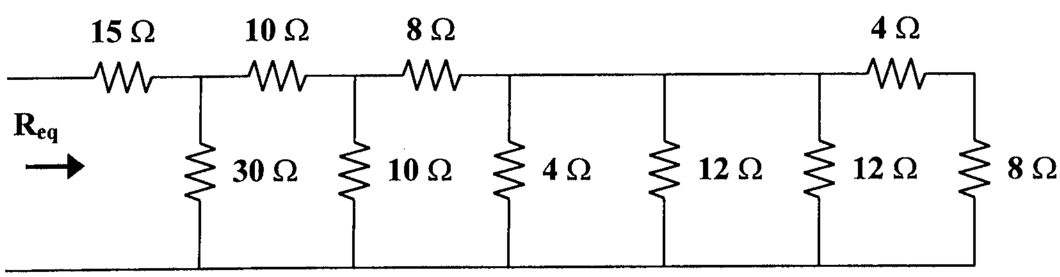
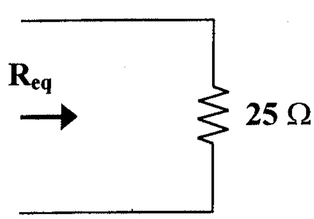


Figure 2.7



R_{eq} = <u>**25 ohms**</u>

Problem 2.15 Using current division, determine i_1 and i_2 in Figure 2.8.

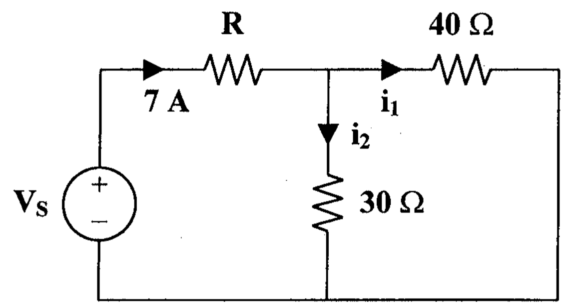


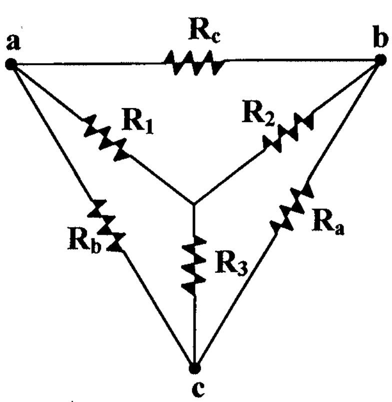
Figure 2.8

$i_1 =$ **3 amps** $\qquad\qquad$ $i_2 =$ **4 amps**

WYE-DELTA TRANSFORMATIONS

The following is a summary of the conversions between wye and delta connected loads.

Given the following resistor network, the Y-Δ equations are listed in the left column and the Δ-Y equations are listed in the right column.



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \qquad R_2 = \frac{R_aR_c}{R_a + R_b + R_c}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \qquad R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$$

Problem 2.16 Find R_{eq} for Figure 2.9.

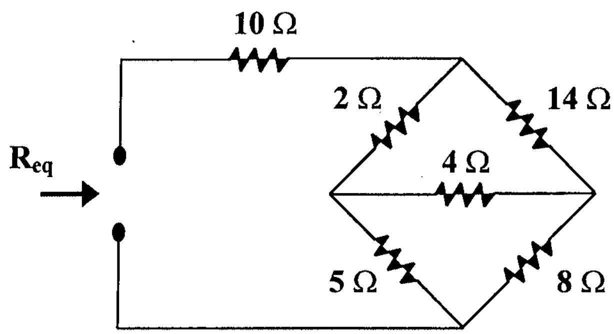


Figure 2.9

- **Carefully DEFINE the problem.**
  Each resistor has a value and the equivalent resistance is shown to be the resistance of the network at the dotted terminals.

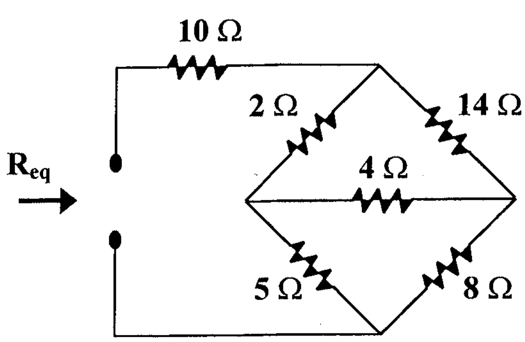
- **PRESENT everything you know about the problem.**
  To find equivalent resistance, the resistor network must be reduced using series combinations, parallel combinations, and conversions between wye and delta connected resistors. We know that series resistances are added to obtain the equivalent resistance and the inverse of parallel resistances are added to obtain the inverse of the equivalent resistance; i.e.

$$R_{eq} = R_1 + R_2 \qquad \text{and} \qquad 1/R_{eq} = 1/R_1 + 1/R_2$$

  We also know how to convert between wye and delta connected loads, as seen previously in this section.

- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
  The set of alternatives to reducing resistor networks involves the various ways that resistors can be combined. In this case, we will convert the lower Δ connection to a Y connection. Then, we will combine parallel resistors (two series resistors are in parallel with two series resistors) to get a series combination. This will produce the equivalent resistance of the resistor network.

- **ATTEMPT a problem solution.**

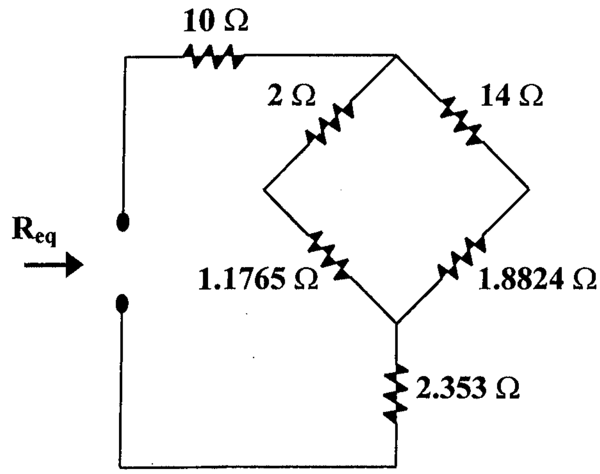


$$R_a + R_b + R_c = 8 + 5 + 4 = 17$$

$$R_1 = \frac{(4)(5)}{17} = 1.1765$$

$$R_2 = \frac{(4)(8)}{17} = 1.8824 \qquad \Longrightarrow$$

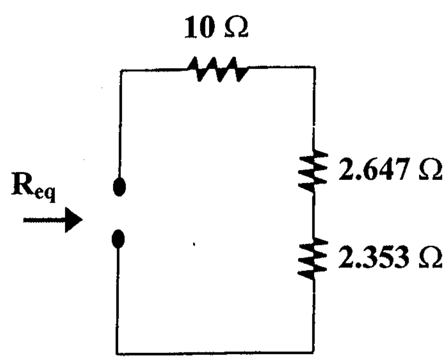
$$R_3 = \frac{(5)(8)}{17} = 2.353$$



$$2 + 1.1765 = 3.177$$
$$14 + 1.8824 = 15.882$$

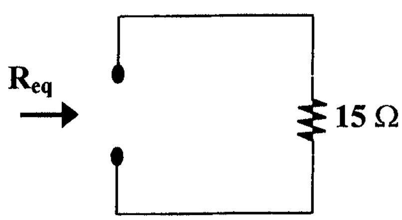
$$3.177 \parallel 15.882$$
$$= \frac{(3.177)(15.882)}{3.177 + 15.882}$$
$$= \frac{50.46}{19.059} = 2.647$$

$\Longrightarrow$



$$10 + 2.647 + 2.353 = 15$$

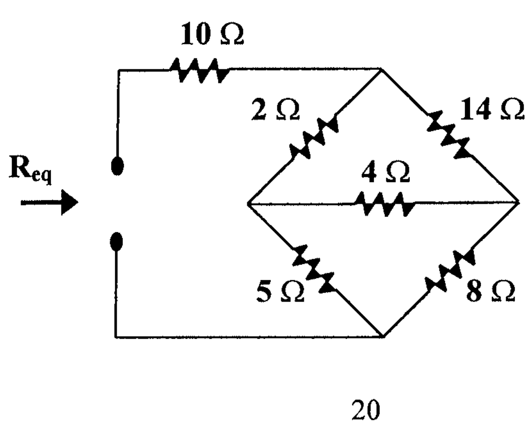
$\Longrightarrow$



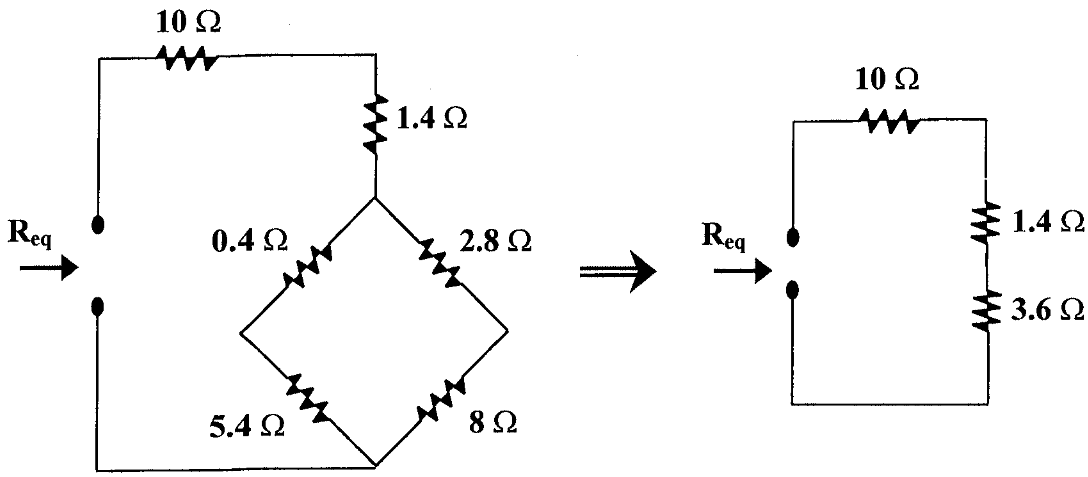
Therefore, $R_{eq} = 15$ ohms.

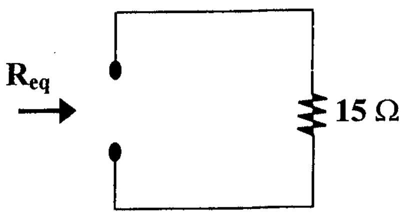
- **EVALUATE the solution and check for accuracy.**
  To check for accuracy, reduce the resistor network by converting the upper Δ connection to a Y connection. Then, combine parallel resistors (two series resistors are in parallel with two series resistors) to get a series combination. This will produce the equivalent resistance of the resistor network.

  It can be shown that



$\Longrightarrow$





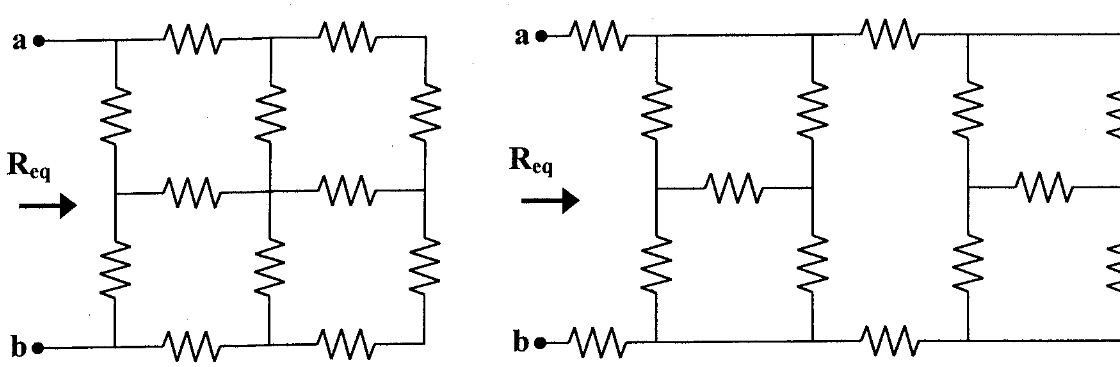
Therefore, $R_{eq} = 15$ ohms.

This is the same answer as that obtained above. Our check for accuracy was successful.

➢ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

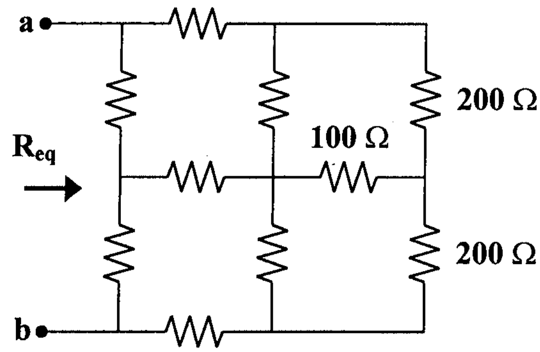
$$R_{eq} = \underline{\mathbf{15\ ohms}}$$

Problem 2.17 *[2.47]* Find the equivalent resistance R_{eq} in each of the following circuits where each resistor has a value of 100 Ω.

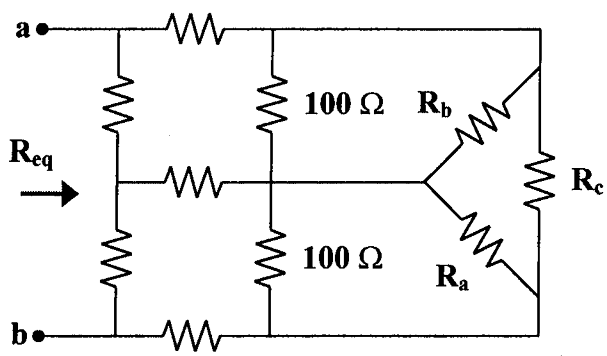


(a) (b)

(a) Begin by combining the series resistors on the rightmost corners of the network.



Now, convert the T (or Y) connection to a Δ connection.

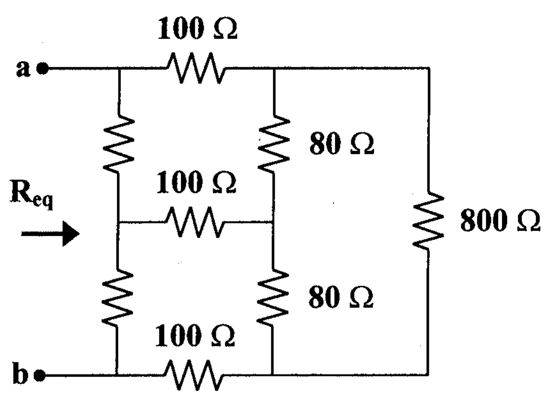


$$R_a = \frac{(200)(200)+(200)(100)+(100)(200)}{200} = \frac{80000}{200} = 400 \text{ ohms}$$

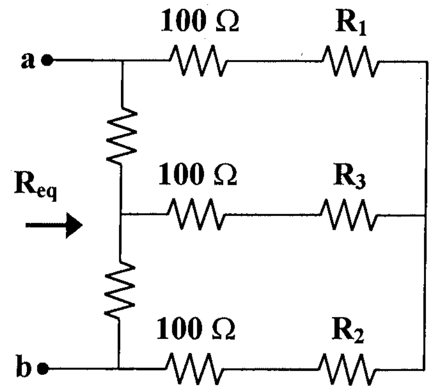
$$R_b = \frac{80000}{200} = 400 \text{ ohms}$$

$$R_c = \frac{80000}{100} = 800 \text{ ohms}$$

Notice that $R_a \| 100$ and $R_b \| 100$ where $400 \| 100 = \dfrac{(400)(100)}{400+100} = \dfrac{40000}{500} = 80$ ohms.

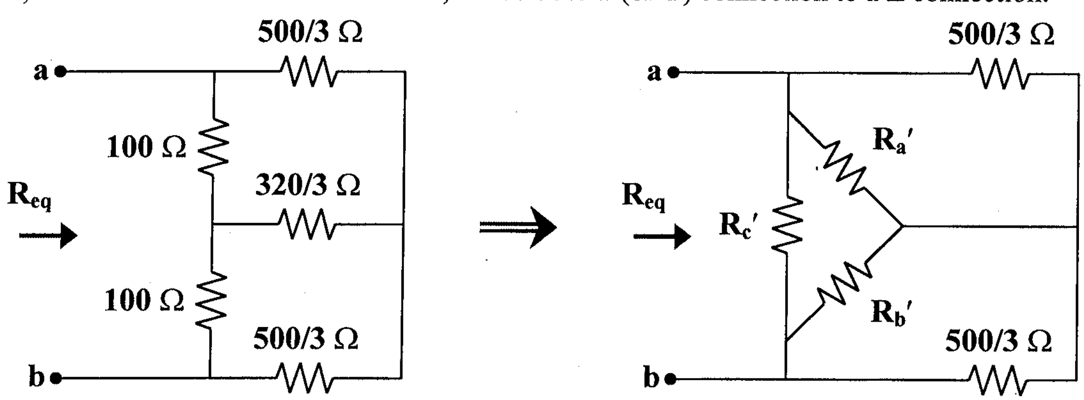


Now, convert Δ connection to a Y connection.



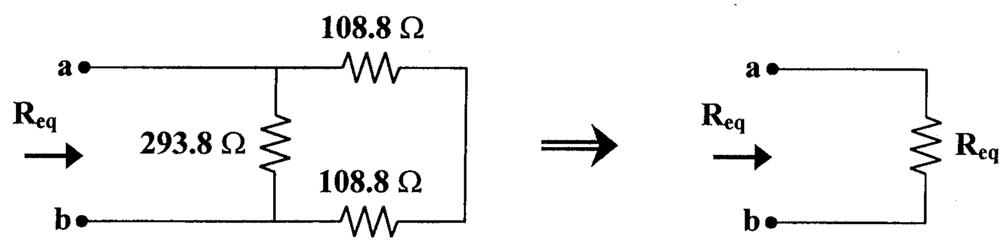
$$R_1 = \frac{(80)(800)}{80+80+800} = \frac{64000}{960} = \frac{200}{3} \text{ ohms}$$
$$R_2 = \frac{(80)(800)}{80+80+800} = \frac{64000}{960} = \frac{200}{3} \text{ ohms}$$
$$R_3 = \frac{(80)(80)}{80+80+800} = \frac{6400}{960} = \frac{20}{3} \text{ ohms}$$

Now, combine the series resistors. Then, convert the T (or Y) connection to a Δ connection.



$$(100)(100) + (100)(320/3) + (320/3)(100) = 94000/3$$
$$R_a' = \frac{94000/3}{100} = \frac{940}{3} = 313.3 \text{ ohms}$$
$$R_b' = \frac{94000/3}{100} = \frac{940}{3} = 313.3 \text{ ohms}$$
$$R_c' = \frac{94000/3}{320/3} = \frac{94000}{320} = 293.8 \text{ ohms}$$

Note that $R_a'\|500/3$ and $R_b'\|500/3$ where $\dfrac{(940/3)(500/3)}{(940/3)+(500/3)} = \dfrac{(940)(500)}{(3)(1440)} = 108.8$ ohms.

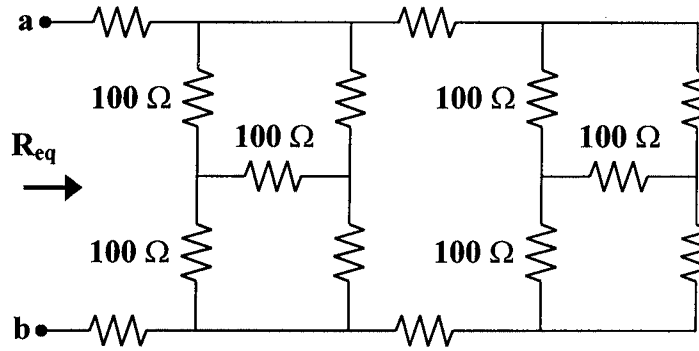


Therefore,

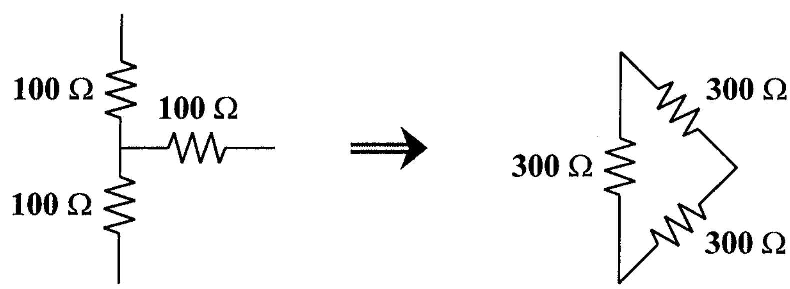
$$R_{eq} = 293.8 \| (108.8 + 108.8) = 293.8 \| 217.6 = \frac{(293.8)(217.6)}{293.8 + 217.6} = \frac{63930}{511.4}$$

$$R_{eq} = \underline{\mathbf{125.01\ ohms}}$$

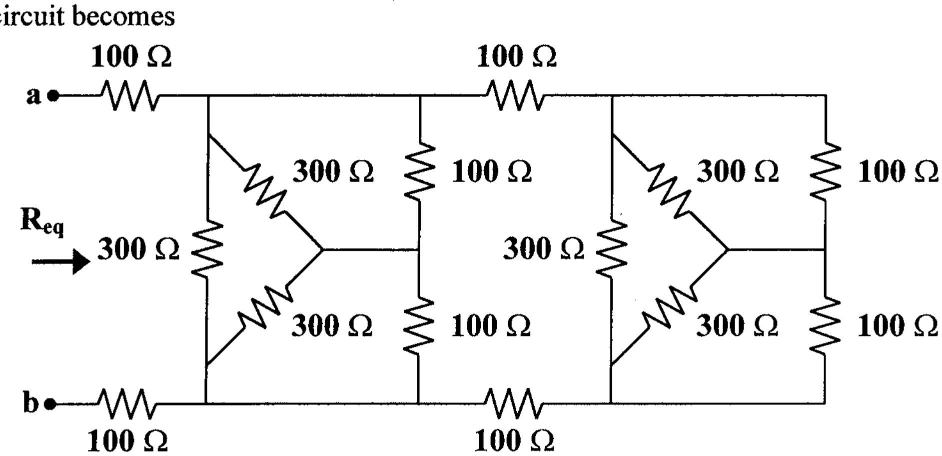
(b) Convert the T connected resistors, labeled in the circuit below, to Δ connected resistors.



It can be shown that



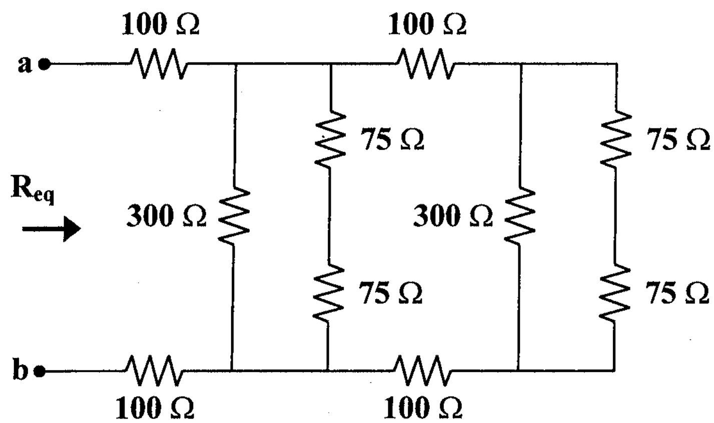
So, the circuit becomes



Now, each of the slanted branches of the Δ connection is in parallel with a 100 Ω resistor.

$$300 \,||\, 100 = \frac{(300)(100)}{300+100} = \frac{30000}{400} = 75 \text{ ohms}$$

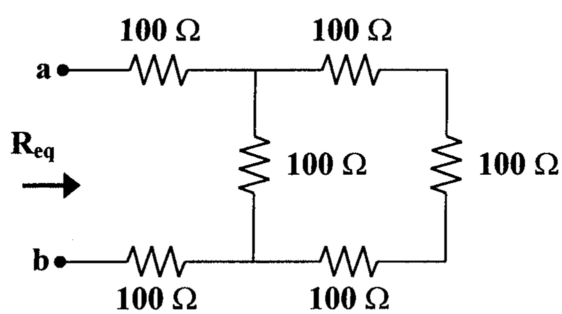
This leads to the following circuit.



Clearly, now we have two places in the circuit where a 300 ohm resistor is in parallel with the series combination of two 75 ohm resistors.

$$300 \,||\, (75+75) = 300 \,||\, 150 = \frac{(300)(150)}{300+150} = \frac{45000}{450} = 100 \text{ ohms}$$

This simplifies the circuit to



It is evident that we have

$$R_{eq} = 100 + 100 \| (100+100+100) + 100$$

or

$$R_{eq} = 100 + 100 \| 300 + 100$$

where we have already shown that $100 \| 300 = 75$.

Therefore,

$$R_{eq} = 100 + 75 + 100$$
$$R_{eq} = \underline{\mathbf{275\ ohms}}$$

APPLICATIONS

Problem 2.18 Given a real voltmeter with an internal resistance of R_m, determine its effect when measuring the voltage across R_2 in Figure 2.10.

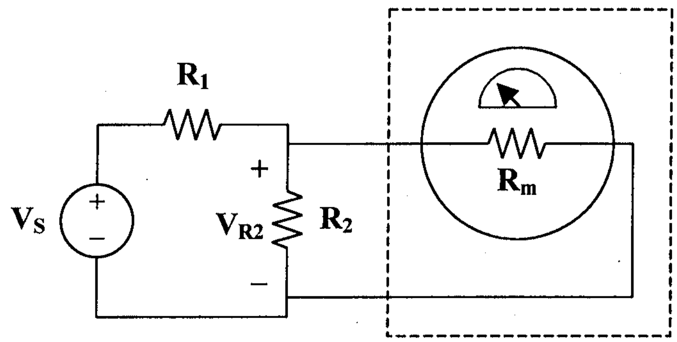


Figure 2.10

- **Carefully DEFINE the problem.**
  Each component is labeled, indicating value and polarity. The problem is clear.

- **PRESENT everything you know about the problem.**
  The voltmeter measures the voltage across a component and is therefore connected in parallel with the component. As shown in the dashed box in Figure 2.10, the voltmeter consists of a d'Arsonval movement in series with a resistor whose internal resistance is deliberately made very large to minimize the current drawn from the circuit. R_m represents the total resistance of the meter.

- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
  To show the effects of the internal resistance of a real voltmeter,
  (a) find V_{R2} if R_m is an open circuit, and
  (b) find V_{R2} if R_m is a short circuit.

- **ATTEMPT a problem solution.**

Using voltage division,

$$V_{R2} = \left[\frac{R_2 \| R_m}{R_1 + (R_2 \| R_m)}\right] V_S$$

(a) $$\lim_{R_m \to \infty} V_{R2} = \lim_{R_m \to \infty}\left[\frac{R_2 \| R_m}{R_1 + R_2 \| R_m}\right] V_S = \lim_{R_m \to \infty}\left[\frac{(\frac{1}{R_2} + \frac{1}{R_m})^{-1}}{R_1 + (\frac{1}{R_2} + \frac{1}{R_m})^{-1}}\right] V_S = \left[\frac{R_2}{R_1 + R_2}\right] V_S$$

(b) $$\lim_{R_m \to 0} V_{R2} = \lim_{R_m \to 0}\left[\frac{R_2 \| R_m}{R_1 + R_2 \| R_m}\right] V_S = \lim_{R_m \to 0}\left[\frac{\left(\frac{R_2 R_m}{R_2 + R_m}\right)}{R_1 + \left(\frac{R_2 R_m}{R_2 + R_m}\right)}\right] V_S = \left[\frac{0}{R_1 + 0}\right] V_S = 0$$

➢ **EVALUATE the solution and check for accuracy.**

(a) If $R_m = \infty$ (open circuit), then all the current flowing through R_1 will continue through R_2. Clearly, using voltage division, $V_{R2} = \left[R_2/(R_1 + R_2)\right] V_S$

(b) If $R_m = 0$ (short circuit), then no current will flow through R_2 and $V_{R2} = 0$. This makes sense due to the fact that current flows through the path of least resistance.

➢ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

<u>If $R_m >> R_2$, the internal resistance will only slightly effect the circuit. As the value of R_m approaches R_2, the effect of the internal resistance becomes increasingly more significant.</u>

Problem 2.19 *[2.55]* As a design engineer, you are asked to design a lighting system consisting of a 70 W power supply and two light bulbs as shown in Figure 2.11. You must select the two bulbs of the following three available bulbs.

(a) $R_1 = 80$ ohms cost = \$0.60 (standard size)

(b) $R_2 = 90$ ohms cost = \$0.90 (standard size)

(c) $R_3 = 100$ ohms cost = \$0.75 (nonstandard size)

The system should be designed for a minimum cost such that $I = 1.2 \text{ A} \pm 5\%$.

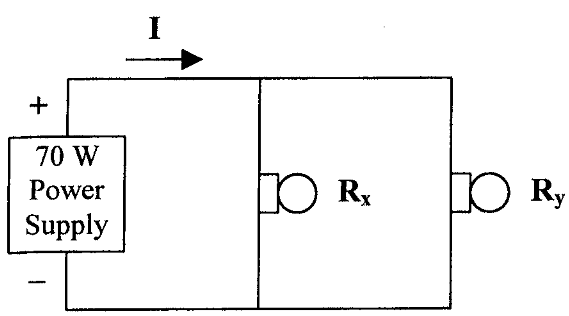


Figure 2.11

Since we need two of the three bulbs, there are only three possibilities.

(a) Use R_1 and R_2.

$$R = R_1 \| R_2 = 80 \| 90 = 42.35 \text{ ohms}$$

$$I = 1.2 \ \pm 5\% = 1.2 \pm 0.06 = 1.26, 1.14 \text{ amps}$$

$$p = I^2 R = \begin{cases} (1.26)^2 (42.35) = 67.23 \text{ W} \\ (1.14)^2 (42.35) = 55.04 \text{ W} \end{cases}$$

$$\text{cost} = \$0.60 + \$0.90 = \$1.50$$

(b) Use R_1 and R_3.

$$R = R_1 \| R_3 = 80 \| 100 = 44.44 \text{ ohms}$$
$$I = 1.2 \ \pm 5\% = 1.2 \pm 0.06 = 1.26, 1.14 \text{ amps}$$
$$p = I^2 R = \begin{cases} (1.26)^2 (44.44) = 70.55 \text{ W} \\ (1.14)^2 (44.44) = 57.75 \text{ W} \end{cases}$$
$$\text{cost} = \$0.60 + \$0.75 = \$1.35$$

(c) Use R_2 and R_3.

$$R = R_2 \| R_3 = 90 \| 100 = 47.37 \text{ ohms}$$
$$I = 1.2 \ \pm 5\% = 1.2 \pm 0.06 = 1.26, 1.14 \text{ amps}$$
$$p = I^2 R = \begin{cases} (1.26)^2 (47.37) = 75.2 \text{ W} \\ (1.14)^2 (47.37) = 61.56 \text{ W} \end{cases}$$
$$\text{cost} = \$0.90 + \$0.75 = \$1.65$$

Note that case (b) represents the lowest cost, however both (b) and (c) have a power that exceeds the 70 W power that can be supplied. Therefore, the correct design uses case (a), i.e.

$\underline{\mathbf{R_1} \textbf{ and } \mathbf{R_2}.}$

Problem 2.20 Given the circuit in Figure 2.12,

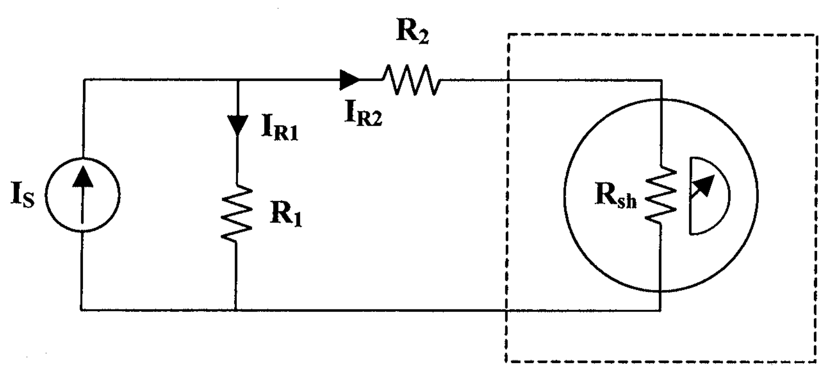


Figure 2.12

find I_{R2} for $I_S = 10$ mA, $R_{sh} = 5\ \Omega$, and

(a) $R_1 = 10\text{ k}\Omega$ and $R_2 = 10\text{ k}\Omega$

(b) $R_1 = 1000\ \Omega$ and $R_2 = 1000\ \Omega$

(c) $R_1 = 50\ \Omega$ and $R_2 = 50\ \Omega$

(d) $R_1 = 5\ \Omega$ and $R_2 = 5\ \Omega$

Using current division,

$$I_{R2} = \left[\frac{R_1}{R_1 + (R_2 + R_{sh})}\right] I_S$$

(a) $$I_{R2} = \left[\frac{10\times10^3}{(10\times10^3)+(10\times10^3)+5}\right](10\times10^{-3}) = \left[\frac{100}{2.001\times10^4}\right] = 4.998\text{ mA}$$

(b) $$I_{R2} = \left[\frac{1\times10^3}{(1\times10^3)+(1\times10^3)+5}\right](10\times10^{-3}) = \left[\frac{10}{2.005\times10^3}\right] = 4.987\text{ mA}$$

(c) $$I_{R2} = \left[\frac{50}{50+50+5}\right](10\times10^{-3}) = \left[\frac{0.5}{105}\right] = 4.762\text{ mA}$$

(d) $$I_{R2} = \left[\frac{5}{5+5+5}\right](10\times10^{-3}) = \left[\frac{0.05}{15}\right] = 3.333\text{ mA}$$

In summary,

(a) $I_{R2} = \underline{\mathbf{4.998\ mA}}$

(b) $I_{R2} = \underline{\mathbf{4.987\ mA}}$

(c) $I_{R2} = \underline{\mathbf{4.762\ mA}}$

(d) $I_{R2} = \underline{\mathbf{3.333\ mA}}$

These answers can be compared to the case where $R_{sh} = 0$ to see just how much the current through R_2 is affected by the internal resistance of the real ammeter. In each case, it can be shown that

$$I_{R2} = \left[\frac{R_1}{R_1 + R_2}\right] I_S = (0.5)(10\times10^{-3}) = 5\text{ mA}$$

CHAPTER 3 - METHODS OF ANALYSIS

List of topics for this chapter :

- Solving Systems of Equations
- Nodal Analysis
- Nodal Analysis with Voltage Sources
- Mesh Analysis
- Mesh Analysis with Current Sources
- Nodal and Mesh Analysis by Inspection
- Circuit Analysis with PSpice

SOLVING SYSTEMS OF EQUATIONS

Problem 3.1 Invert a general $n \times n$ matrix.

The inverse of a nonsingular $n \times n$ matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

is given by

$$A^{-1} = \frac{C^T}{\Delta} = \frac{1}{\Delta} \begin{bmatrix} c_{11} & c_{21} & \cdots & c_{n1} \\ c_{12} & c_{22} & \cdots & c_{n2} \\ \vdots & \vdots & \cdots & \vdots \\ c_{1n} & c_{2n} & \cdots & c_{nn} \end{bmatrix}$$

where Δ is the determinant of the matrix A and c_{ij} is the cofactor of a_{ij} in Δ.

The value of the determinant, Δ, can be obtained by expanding along the *i*th row

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = (-1)^{i+1} a_{i1}\, m_{i1} + (-1)^{i+2} a_{i2}\, m_{i2} + \cdots + (-1)^{i+n} a_{in}\, m_{in}$$

or the *j*th column

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = (-1)^{1+j} a_{1j}\, m_{1j} + (-1)^{2+j} a_{2j}\, m_{2j} + \cdots + (-1)^{n+j} a_{nj}\, m_{nj}$$

where m_{ij}, the minor of a_{ij} in Δ, is a $(n-1)\times(n-1)$ determinant of the submatrix of A obtained by removing the *i*th row and the *j*th column.

The cofactor of a_{ij} in Δ is $\qquad c_{ij} = (-1)^{i+j}\, m_{ij}$.

The transpose of the cofactor matrix is also known as the adjoint of the matrix; i.e., $\mathrm{adj}\,A = C^T$.

Therefore, $$\underline{A^{-1} = \frac{\mathbf{adj\,A}}{\mathbf{det\,A}} = \frac{C^T}{\Delta}}$$

Problem 3.2 Solve a general system of simultaneous equations using Cramer's rule.

Given a system of simultaneous equations having the form

$$\begin{aligned} a_{11}\,x_1 + a_{12}\,x_2 + \cdots + a_{1n}\,x_n &= b_1 \\ a_{21}\,x_1 + a_{22}\,x_2 + \cdots + a_{2n}\,x_n &= b_2 \\ \vdots \qquad \vdots \qquad\quad \vdots \qquad & \vdots \\ a_{n1}\,x_1 + a_{n2}\,x_2 + \cdots + a_{nn}\,x_n &= b_n \end{aligned}$$

where there are n unknowns, x_1, x_2, ..., x_n, to be determined.

The matrix representation of the system of simultaneous equations is

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

or $$AX = B$$

where $$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Note that A is a square $(n \times n)$ matrix, while X and B are $(n \times 1)$ column matrices.

Cramer's rule states that the solution to the system of simultaneous equations, $AX = B$, is

$$x_1 = \frac{\Delta_1}{\Delta}, \quad x_2 = \frac{\Delta_2}{\Delta}, \ldots, \quad x_n = \frac{\Delta_n}{\Delta}$$

where the Δ's are the determinants given by

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix} \quad \Delta_2 = \begin{vmatrix} a_{11} & b_1 & \cdots & a_{1n} \\ a_{21} & b_2 & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & b_n & \cdots & a_{nn} \end{vmatrix} \quad \cdots \quad \Delta_n = \begin{vmatrix} a_{11} & a_{12} & \cdots & b_1 \\ a_{21} & a_{22} & \cdots & b_2 \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & b_n \end{vmatrix}$$

Notice that Δ is the determinant of matrix A and Δ_k is Δ with its *k*th column replaced with matrix B.

Obviously, Cramer's rule only applies when $\Delta \neq 0$. In the case that $\Delta = 0$, the set of equations has no unique solution because the equations are linearly dependent.

See Problem 3.1 to find out how to calculate the value of a determinant of a matrix.

NODAL ANALYSIS

Problem 3.3 *[3.3]* Find the currents i_1 through i_4 and the voltage v_o in Figure 3.1.

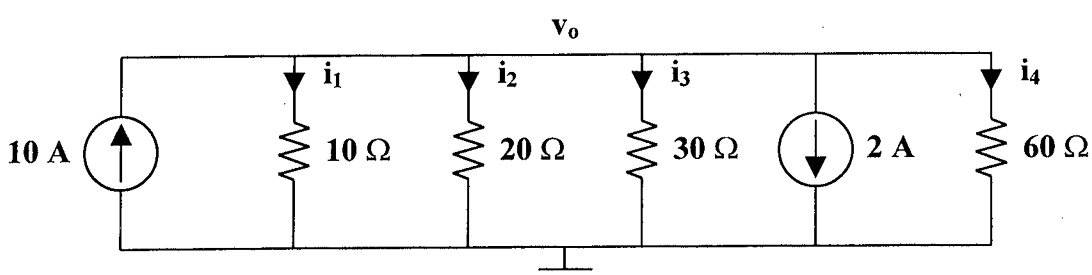


Figure 3.1

Applying KCL to the non-reference node,

$$10 = \frac{v_o}{10} + \frac{v_o}{20} + \frac{v_o}{30} + 2 + \frac{v_o}{60}$$

$$480 = 12\,v_o \longrightarrow v_o = \underline{\mathbf{40\ V}}$$

Thus,

$$i_1 = \frac{v_o}{10} \qquad i_2 = \frac{v_o}{20} \qquad i_3 = \frac{v_o}{30} \qquad i_4 = \frac{v_o}{60}$$

$$i_1 = \underline{\mathbf{4\ A}} \qquad i_2 = \underline{\mathbf{2\ A}} \qquad i_3 = \underline{\mathbf{1.3333\ A}} \qquad i_4 = \underline{\mathbf{666.7\ mA}}$$

Problem 3.4 Given the circuit in Figure 3.2, find I_x.

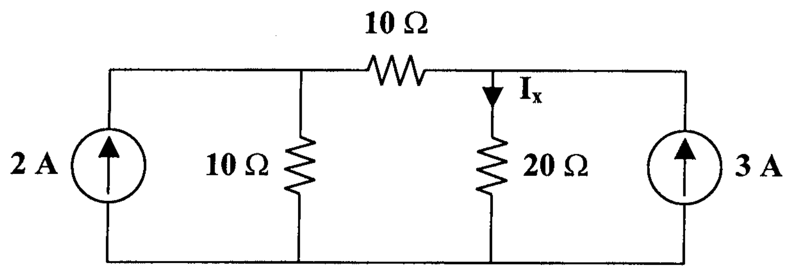


Figure 3.2

$$I_x = \underline{\mathbf{2\ A}}$$

NODAL ANALYSIS WITH VOLTAGE SOURCES

Problem 3.5 Given the circuit in Figure 3.3, solve for V_x.

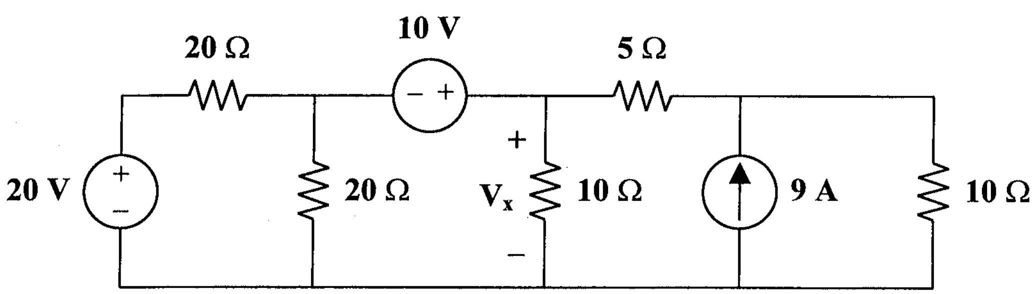


Figure 3.3

➢ **Carefully DEFINE the problem.**
Each component is labeled completely. The problem is clear.

➢ **PRESENT everything you know about the problem.**
The goal of the problem is to find V_x. Letting the lower node be the reference node, we need to find the voltage to the right of the 10-V voltage source.

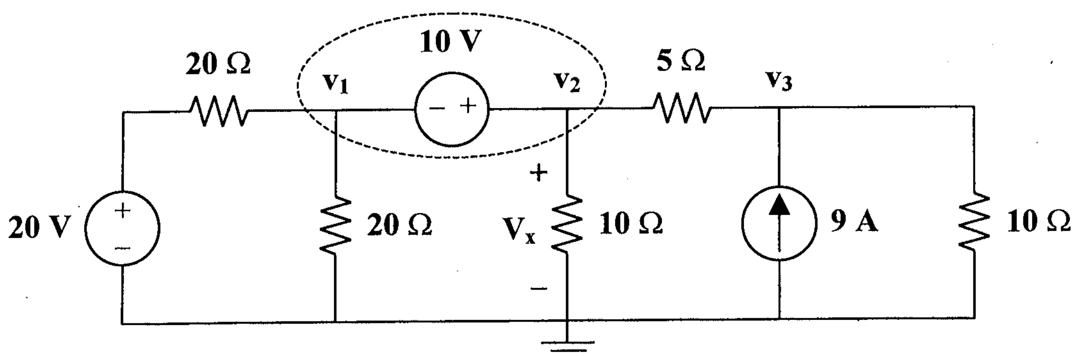
A supernode is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it. Hence, it is clear that the two nodes on either side of the 10-V voltage source form a supernode.

A supermesh results when two meshes have a (dependent or independent) current source in common. Hence, the two meshes on the right half of the circuit create a supermesh, with the 9-A current source in common.

➢ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
The two methods of analysis of simple circuits, such as the one above, are nodal analysis and mesh analysis. Because the circuit contains three nodes in addition to the reference node, nodal analysis produces a set of three equations and three unknowns. Yet, the supernode changes the set to two equations and a constraint equation. On the other hand, the circuit has four loops. Thus, mesh analysis produces four equations and four unknowns. The supermesh changes the set to three equations and a constraint equation.

Because the goal of the problem is to find a voltage, the obvious choice is to use nodal analysis to find the voltages at each node of the circuit. If mesh analysis were used to find the mesh currents, Ohm's law would also be needed to find the voltage across the resistor.

➢ **ATTEMPT a problem solution.**
Begin the problem solution by identifying the nodes, including the supernode.



Clearly, $V_x = v_2$

Use nodal analysis to find v_1, v_2, and v_3.
At the supernode (nodes 1 & 2):

$$\frac{v_1 - 20}{20} + \frac{v_1 - 0}{20} + \frac{v_2 - 0}{10} + \frac{v_2 - v_3}{5} = 0$$

At node 3:

$$\frac{v_3 - v_2}{5} - 9 + \frac{v_3 - 0}{10} = 0$$

Simplifying,

$$(v_1 - 20) + v_1 + 2v_2 + (4)(v_2 - v_3) = 0$$
$$2v_1 + 6v_2 - 4v_3 = 20$$
$$v_1 + 3v_2 - 2v_3 = 10$$

$$(2)(v_3 - v_2) + v_3 = 90$$
$$-2v_2 + 3v_3 = 90$$

This is a set of two equations and three unknowns. Thus, we must find a constraint equation. The supernode will provide the constraint equation.

$$v_2 = v_1 + 10 \qquad \text{or} \qquad v_1 = v_2 - 10$$

Substitute the constraint equation into the simplified equation from the supernode. Then, this equation plus two times the simplified equation from node 3 will isolate v_3.

$$(v_2 - 10) + 3v_2 - 2v_3 = 10$$
$$(2)[-2v_2 + 3v_3 = 90]$$

$$[4v_2 - 2v_3 = 20] + [-4v_2 + 6v_3 = 180] = [4v_3 = 200]$$
$$v_3 = 50\,\text{volts}$$

The equation at node 3 can be written,

$$2v_2 = 3v_3 - 90$$
$$v_2 = (1/2)[(3)(50) - 90] = (1/2)(150 - 90) = (1/2)(60) = 30\,\text{volts}$$

The constraint equation gives

$$v_1 = v_2 - 10 = 30 - 10 = 20\,\text{volts}$$

Therefore, $\qquad V_x = v_2 = 30\,\text{volts}$

➢ **EVALUATE the solution and check for accuracy.**
This circuit can be analyzed using mesh analysis to verify the solution. This would provide practice analyzing a circuit with a supermesh. Mesh analysis will be discussed later in this chapter. So, we will check our solution using KCL at each node.

For the supernode,

$$\frac{v_1 - 20}{20} + \frac{v_1}{20} + \frac{v_2}{10} + \frac{v_2 - v_3}{5} = \frac{20 - 20}{20} + \frac{20}{20} + \frac{30}{10} + \frac{30 - 50}{5} = 0 + 1 + 3 - 4 = 0$$

For node 3,

$$\frac{v_3 - v_2}{5} - 9 + \frac{v_3}{10} = \frac{50 - 30}{5} - 9 + \frac{50}{10} = 4 - 9 + 5 = 0$$

KCL is not violated. Thus, our check for accuracy was successful.

➢ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

$$V_x = \underline{30\ V}$$

Problem 3.6 *[3.7]* Using nodal analysis, find V_o in Figure 3.4.

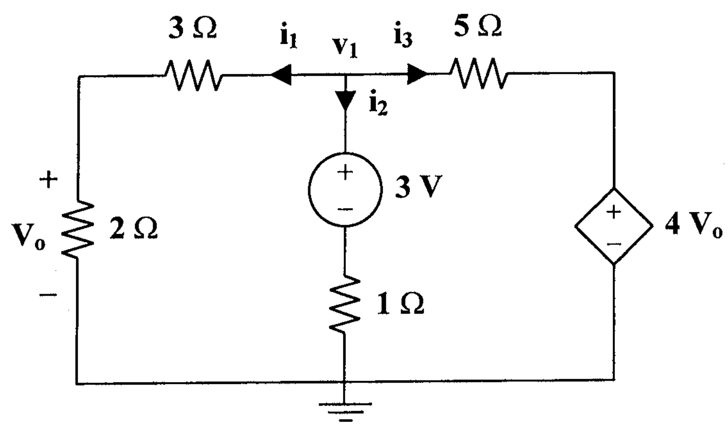


Figure 3.4

$$i_1 + i_2 + i_3 = 0 \longrightarrow \frac{v_1}{5} + \frac{v_1 - 3}{1} + \frac{v_1 - 4V_o}{5} = 0$$

$$7v_1 - 4V_o = 15$$

But $\quad V_o = \frac{2}{5}v_1 \quad$ or $\quad v_1 = \frac{5}{2}V_o$

So that $\quad (7)\left(\frac{5}{2}\right)V_o - 4V_o = 15 \longrightarrow V_o = \frac{(2)(15)}{27}$

$$V_o = \underline{1.1111\ V}$$

Problem 3.7 Given the circuit in Figure 3.5, solve for V_x using matrix inversion.

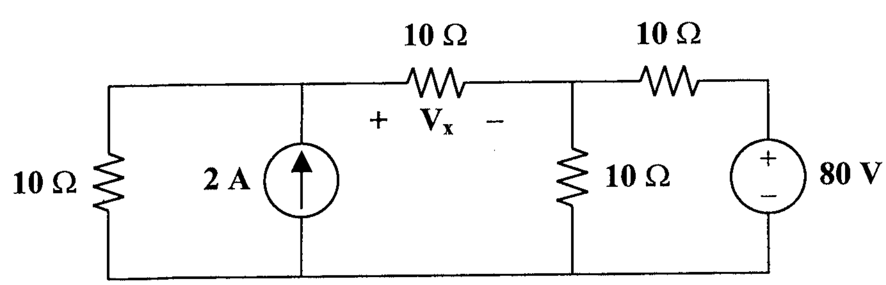
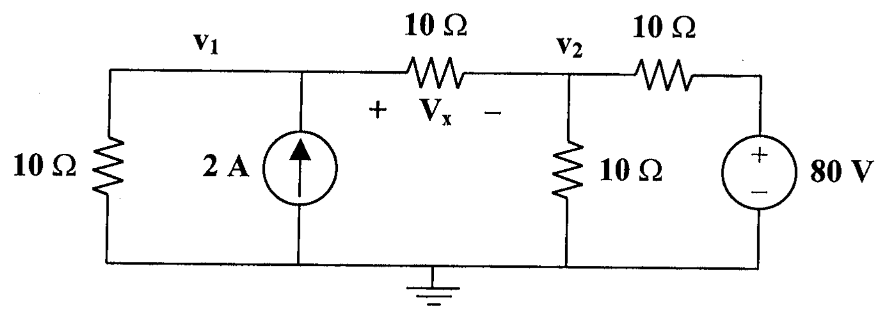


Figure 3.5



Clearly, $\quad v_1 = V_x + v_2 \quad$ or $\quad V_x = v_1 - v_2$

Use nodal analysis to find v_1 and v_2.

At node 1 :

$$\frac{v_1 - 0}{10} - 2 + \frac{v_1 - v_2}{10} = 0$$

At node 2 :

$$\frac{v_2 - 80}{10} + \frac{v_2 - 0}{10} + \frac{v_2 - v_1}{10} = 0$$

Simplifying,

$$v_1 - 20 + v_1 - v_2 = 0$$
$$2v_1 - v_2 = 20$$

$$v_2 - 80 + v_2 + v_2 - v_1 = 0$$
$$-v_1 + 3v_2 = 80$$

The system of simultaneous equations is

$$\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 80 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{6-1}\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 20 \\ 80 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 60+80 \\ 20+160 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 140 \\ 180 \end{bmatrix} = \begin{bmatrix} 28 \\ 36 \end{bmatrix}$$

Therefore, $\quad V_x = v_1 - v_2 = 28 - 36$

$$V_x = \underline{\mathbf{-8\ V}}$$

Problem 3.8 Solve Problem 3.5 using Cramer's rule.

$$V_x = \underline{\mathbf{30\ V}}$$

This answer is the same as that found in Problem 3.5.

Problem 3.9 Given the circuit in Figure 3.6, solve for the loop currents, i_1 and i_2, using mesh analysis.

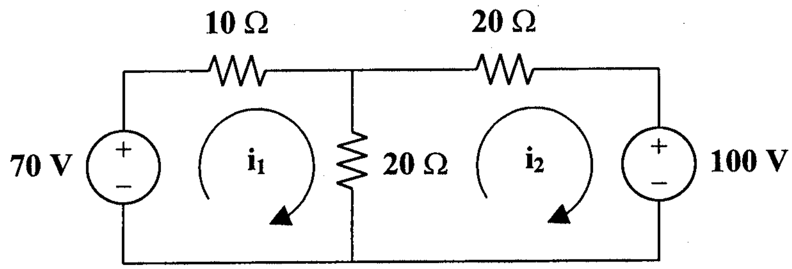


Figure 3.6

For loop 1 :

$$-70 + 10i_1 + (20)(i_1 - i_2) = 0$$
$$30i_1 - 20i_2 = 70$$
$$3i_1 - 2i_2 = 7$$

For loop 2 :

$$(20)(i_2 - i_1) + 20i_2 + 100 = 0$$
$$-20i_1 + 40i_2 = -100$$
$$-2i_1 + 4i_2 = -10$$

Thus, the system of simultaneous equations is

$$\begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 7 \\ -10 \end{bmatrix} \text{ or } \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{1}{12-4}\begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 7 \\ -10 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 8 \\ -16 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Therefore, $\underline{i_1 = \mathbf{1\ A}}$ and $\underline{i_2 = \mathbf{-2\ A}}$

Problem 3.10 *[3.33]* Apply mesh analysis to find i in Figure 3.7.

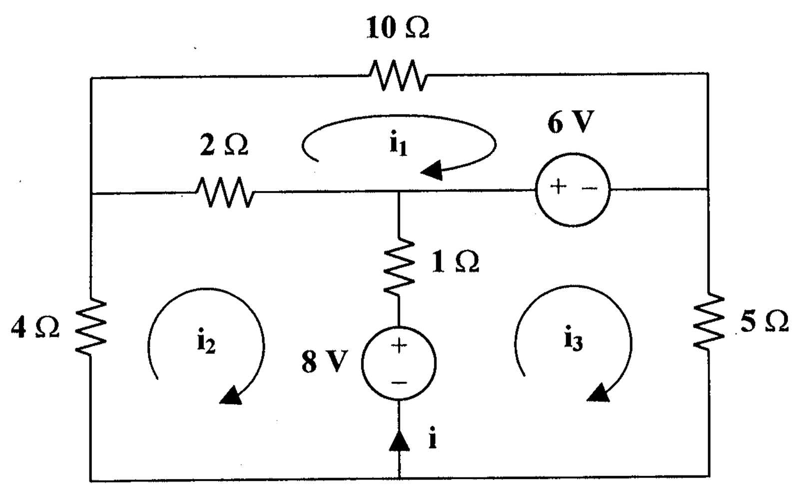


Figure 3.7

For loop 1 : $6 = 12i_1 - 2i_2 \longrightarrow 3 = 6i_1 - i_2$ (1)

For loop 2 : $-8 = 7i_2 - 2i_1 - i_3 \longrightarrow 8 = 2i_1 - 7i_2 + i_3$ (2)

For loop 3 : $-8 + 6 + 6i_3 - i_2 = 0 \longrightarrow 2 = 6i_3 - i_2$ (3)

Putting (1), (2), and (3) in matrix form,

$$\begin{bmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{vmatrix} = -234 \qquad \Delta_2 = \begin{vmatrix} 6 & 3 & 0 \\ 2 & 8 & 1 \\ 0 & 2 & 6 \end{vmatrix} = 240 \qquad \Delta_3 = \begin{vmatrix} 6 & -1 & 3 \\ 2 & -7 & 8 \\ 0 & -1 & 2 \end{vmatrix} = -38$$

Clearly,

$$i = i_3 - i_2 = \frac{\Delta_3 - \Delta_2}{\Delta} = \frac{-38 - 240}{-234}$$

$$i = \underline{\mathbf{1.188\ A}}$$

MESH ANALYSIS WITH CURRENT SOURCES

Problem 3.11 Given the circuit shown in Figure 3.8, find I_x using mesh analysis.

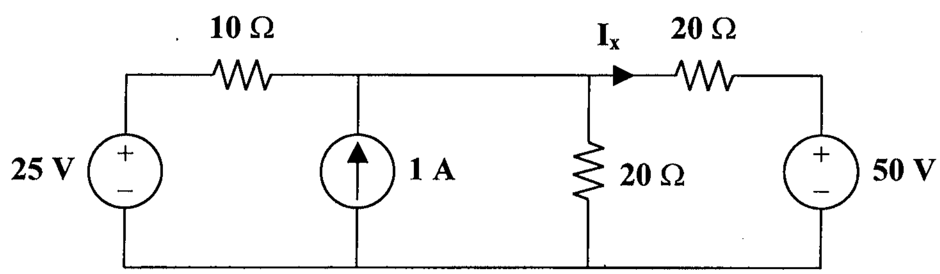


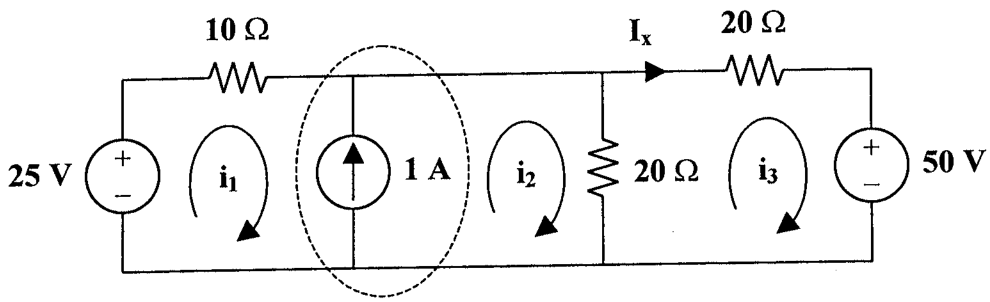
Figure 3.8

- **Carefully DEFINE the problem.**
  Each component is labeled completely. The problem is clear.

- **PRESENT everything you know about the problem.**
  A supermesh results when two meshes have a (dependent or independent) current source in common. Hence, the leftmost mesh and the middle mesh create a supermesh, with the 1-A current source in common.

- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
  The problem clearly states to use mesh analysis. This makes sense because the goal of the problem is to find a current, I_x, and mesh analysis produces the currents in each mesh, or loop, of a circuit.

- **ATTEMPT a problem solution.**
  Begin the problem solution by identifying the meshes, including the supermesh.



Clearly, $I_x = i_3$

Use mesh analysis to find i_1, i_2, and i_3.

For the supermesh (loops 1 & 2): $-25 + 10i_1 + (20)(i_2 - i_3) = 0$

where $1 = i_2 - i_1$ or $i_1 = i_2 - 1$ (constraint equation)

For loop 3 : $50 + (20)(i_3 - i_2) + 20i_3 = 0$

Substitute the constraint equation into the equation for the supermesh and simplify,

$$-25 + 10i_2 - 10 + 20i_2 - 20i_3 = 0$$
$$50 + 20i_3 - 20i_2 + 20i_3 = 0$$

Simplifying further,

$$30i_2 - 20i_3 = 35 \qquad \text{or} \qquad 6i_2 - 4i_3 = 7$$
$$-20i_2 + 40i_3 = -50 \qquad\qquad -2i_2 + 4i_3 = -5$$

The system of simultaneous equations is

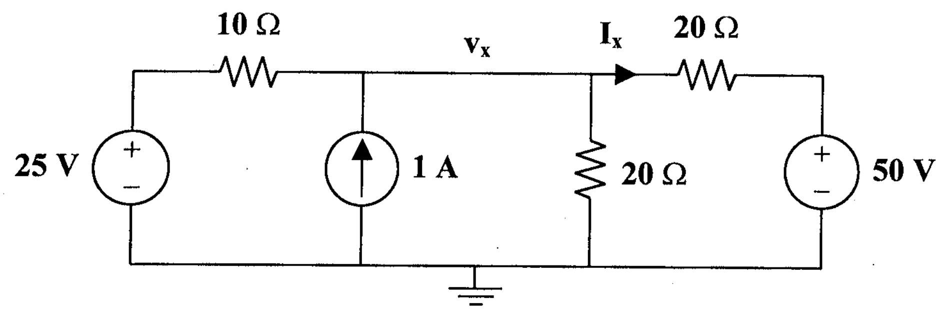
$$\begin{bmatrix} 6 & -4 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \frac{1}{24-8}\begin{bmatrix} 4 & 4 \\ 2 & 6 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 8 \\ -16 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -1 \end{bmatrix}$$

and $i_1 = i_2 - 1 = 0.5 - 1 = -0.5$

Therefore, $I_x = i_3 = -1\,\text{amp}$

➢ **EVALUATE the solution and check for accuracy.**
This circuit has only one unknown node after identifying the lower node as the reference node. Hence, it can easily be analyzed using nodal analysis.



$$\frac{v_x - 25}{10} - 1 + \frac{v_x}{20} + \frac{v_x - 50}{20} = 0$$

$$4\,v_x = 120$$

$$v_x = 30\,\text{volts}$$

Using Ohm's law,

$$I_x = \frac{v_x - 50}{20} = \frac{30 - 50}{20} = \text{-1 amp}$$

This matches the answer that was obtained using mesh analysis. Our check for accuracy was successful.

➢ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

$$I_x = \underline{\mathbf{-1\ A}}$$

Problem 3.12 *[3.35]* Use mesh analysis to obtain i_o in the circuit of Figure 3.9.

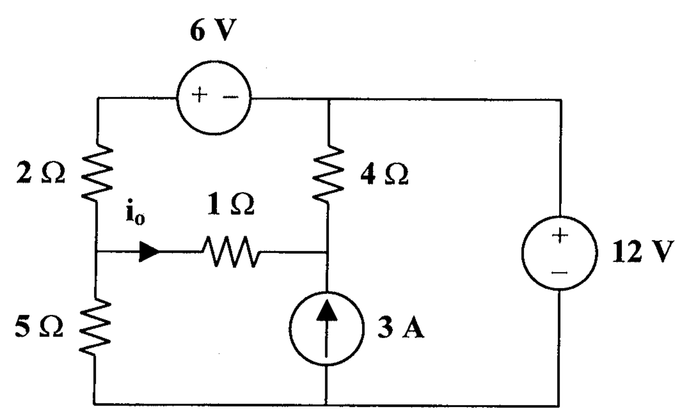
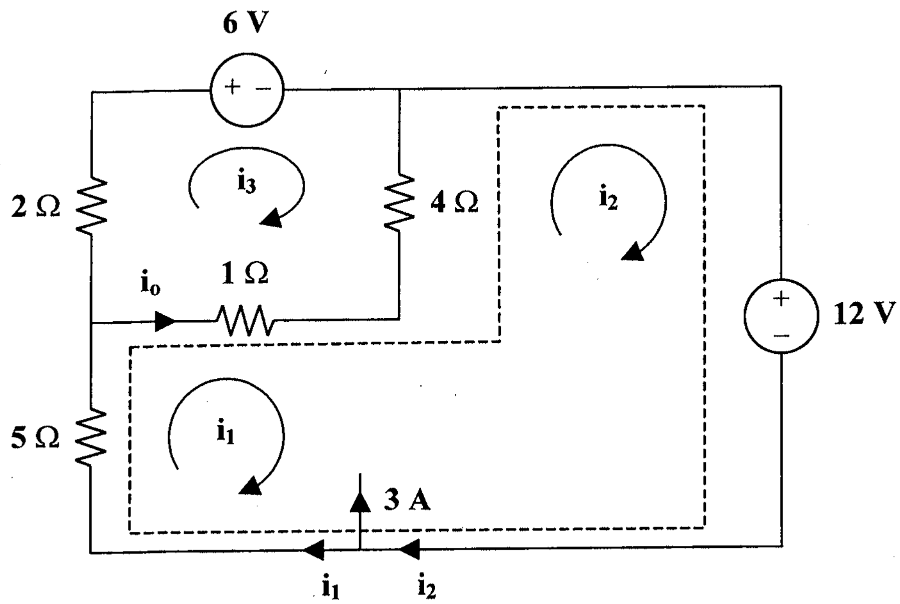


Figure 3.9



Loops 1 and 2 form a supermesh. For the supermesh,

$$6i_1 + 4i_2 - 5i_3 + 12 = 0 \qquad (1)$$

For loop 3,

$$6 + 7i_3 - i_1 - 4i_2 = 0 \qquad (2)$$

Also, $$i_2 = 3 + i_1 \qquad (3)$$

Putting (1), (2), and (3) into matrix form,

$$\begin{bmatrix} 6 & 4 & -5 \\ 1 & 4 & -7 \\ -1 & 1 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 6 \\ 3 \end{bmatrix}$$

Solving this system of equations yields

$$i_1 = -3.067 \text{ amps}$$
$$i_2 = -0.06667 \text{ amps}$$
$$i_3 = -1.3333 \text{ amps}$$

Therefore,

$$i_o = i_1 - i_3 = -3.067 - (-1.3333)$$
$$i_o = \underline{\mathbf{-1.7333\ A}}$$

Problem 3.13 Given the circuit as shown in Figure 3.10, solve for I_x using mesh analysis.

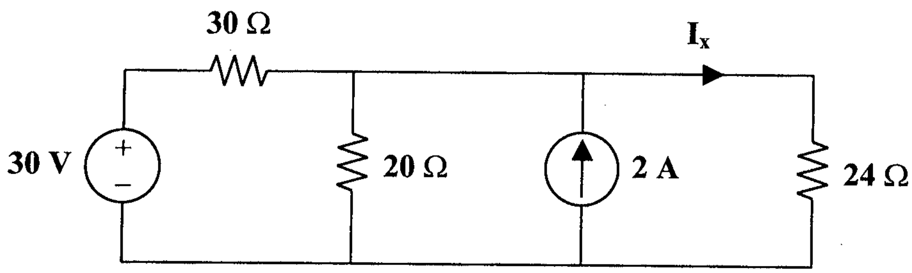


Figure 3.10

$$I_x = \underline{1\ \text{A}}$$

NODAL AND MESH ANALYSIS BY INSPECTION

Problem 3.14 *[3.51]* Obtain the node-voltage equations for the circuit shown in Figure 3.11 by inspection. Determine the node voltages v_1 and v_2.

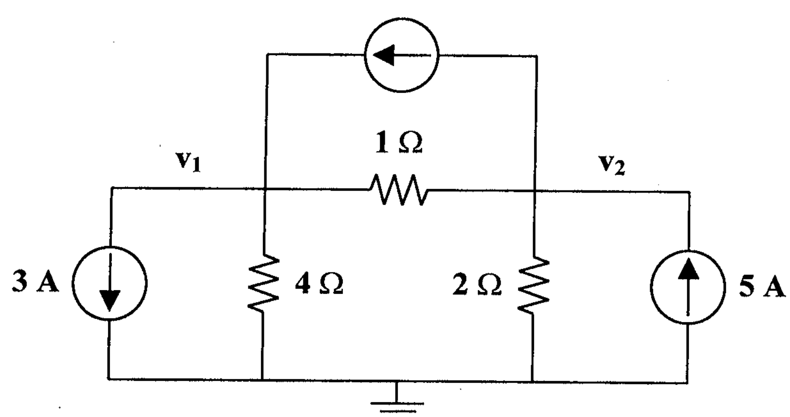


Figure 3.11

$G_{11} = \dfrac{1}{1} + \dfrac{1}{4} = 1.25$ $\quad G_{12} = -1 = G_{21}$ $\quad G_{22} = \dfrac{1}{1} + \dfrac{1}{2} = 1.5$

$i_1 = 6 - 3 = 3$ $\quad i_2 = 5 - 6 = -1$

Hence, we have

$$\begin{bmatrix} \mathbf{1.25} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{1.5} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \mathbf{3} \\ \mathbf{-1} \end{bmatrix}$$

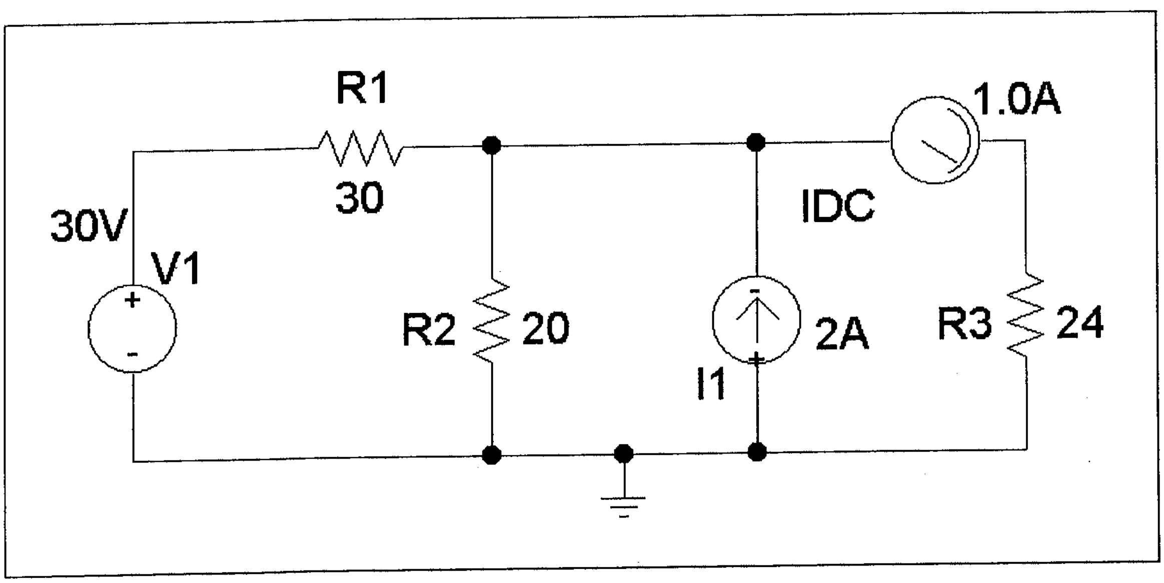
$$\begin{bmatrix} 1.25 & -1 \\ -1 & 1.5 \end{bmatrix}^{-1} = \frac{1}{\Delta} \begin{bmatrix} 1.5 & 1 \\ 1 & 1.25 \end{bmatrix} \qquad \text{where} \quad \Delta = (1.25)(1.5) - (-1)(-1) = 0.875$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1.7143 & 1.1429 \\ 1.1429 & 1.4286 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} (3)(1.7143) + (-1)(1.1429) \\ (3)(1.1429) + (-1)(1.4286) \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

Clearly, $v_1 = \underline{\mathbf{4\ V}}$ and $v_2 = \underline{\mathbf{2\ V}}$

CIRCUIT ANALYSIS WITH PSPICE

Problem 3.15 Solve Problem 3.13 using PSpice.



Clearly, I_x is the current flowing through R3 and the current probe reads $I_x = \underline{\mathbf{1.0\ A}}$.

This answer is the same as the answer obtained in Problem 3.13.

CHAPTER 4 - CIRCUIT THEOREMS

List of topics for this chapter :
- Linearity Property
- Principle of Superposition
- Source Transformations
- Thevenin's Theorem
- Norton's Theorem
- Maximum Power Transfer
- Verifying Circuit Theorems with PSpice

LINEARITY PROPERTY

Linearity is the condition in which the change in value of one quantity is directly proportional to that of another quantity. A linear circuit is one whose output is linearly related (or directly proportional) to its input.

Problem 4.1 If all the independent sources are multiplied by a value, K, and all the currents and voltages of the circuit increase by the same value, then a circuit is linear. Show that the circuit in Figure 4.1 is linear by solving for I_x and V_x.

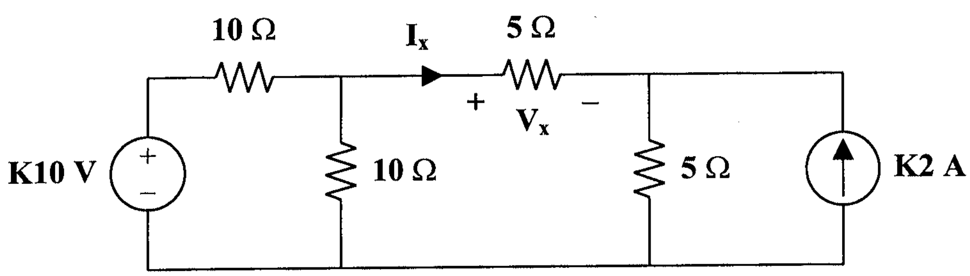
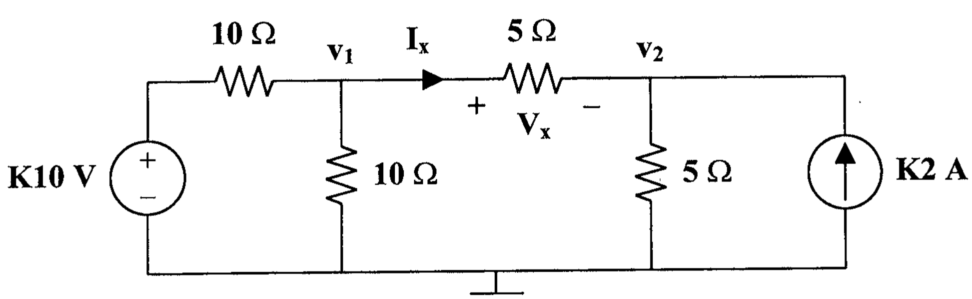


Figure 4.1



Use nodal analysis to find v_1 and v_2.

At node 1 :

$$\frac{v_1 - K10}{10} + \frac{v_1 - 0}{10} + \frac{v_1 - v_2}{5} = 0$$

At node 2 :

$$\frac{v_2 - v_1}{5} + \frac{v_2 - 0}{5} = K2$$

Simplifying,

$$v_1 - K10 + v_1 + (2)(v_1 - v_2) = 0$$
$$4v_1 - 2v_2 = K10$$

$$v_2 - v_1 + v_2 = K10$$
$$-v_1 + 2v_2 = K10$$

The system of simultaneous equations is

$$\begin{bmatrix} 4 & -2 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} K10 \\ K10 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{8-2}\begin{bmatrix} 2 & 2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} K10 \\ K10 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} K20 + K20 \\ K10 + K40 \end{bmatrix} = \begin{bmatrix} (40/6)K \\ (50/6)K \end{bmatrix} = \begin{bmatrix} (20/3)K \\ (25/3)K \end{bmatrix}$$

Clearly, $V_x = v_1 - v_2 = (20/3)K - (25/3)K = (-5/3)K$

and $I_x = V_x/5 = [(-5/3)K](1/5) = (-1/3)K$

The voltage source and current source are multiples of K. The voltage V_x and the current I_x are multiples of K. Therefore, **the circuit is linear**.

Problem 4.2 *[4.3]*

(a) In the circuit in Figure 4.2, calculate v_o and i_o when $v_s = 1$ V.

(b) Find v_o and i_o when $v_s = 10$ V.

(c) What are v_o and i_o when each of the 1-Ω resistors is replaced by a 10-Ω resistor and $v_s = 10$ V?

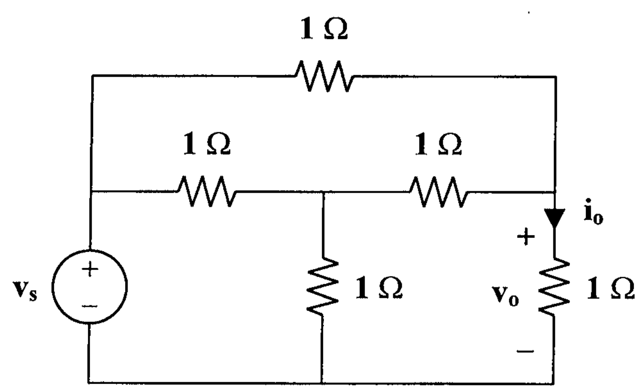
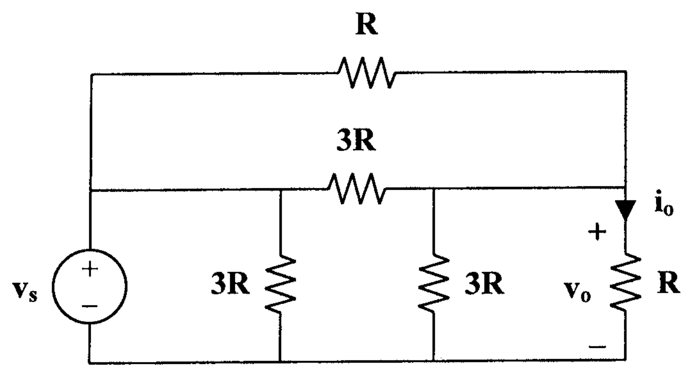
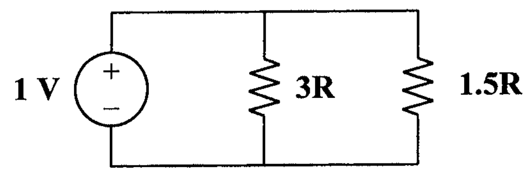


Figure 4.2

First, transform the Y (or T) subcircuit to its Δ (or Π) equivalent.



(a)



(b)

$$R \,||\, 3R = \frac{3R^2}{4R} = \frac{3}{4}R \qquad \frac{3}{4}R + \frac{3}{4}R = \frac{3}{2}R$$

$v_o = v_s/2$, independent of R

and

$i_o = v_o/R$

(a) When $v_s = 1$ V and R = 1 Ω,
$v_o =$ **0.5 V** $\qquad i_o =$ **0.5 A**

(b) When $v_s = 10$ V and R = 1 Ω,
$v_o =$ **5 V** $\qquad i_o =$ **5 A**

(c) When $v_s = 10$ V and $R = 10\,\Omega$,
$v_o =$ **5 V** $\qquad i_o =$ **0.5 A**

PRINCIPLE OF SUPERPOSITION

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

Problem 4.3 Solve for I_x in Figure 4.3 using superposition.

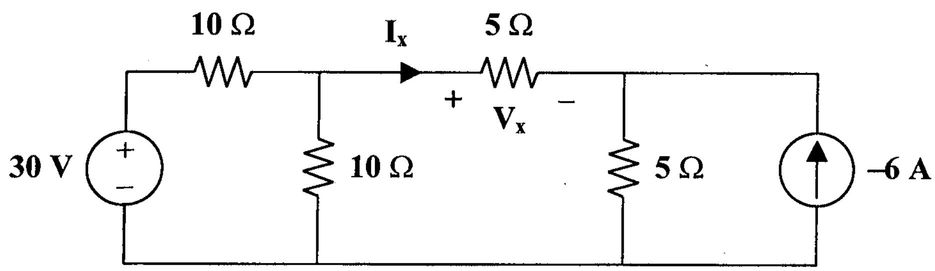


Figure 4.3

- **Carefully DEFINE the problem.**
  Each component is labeled completely. The problem is clear.

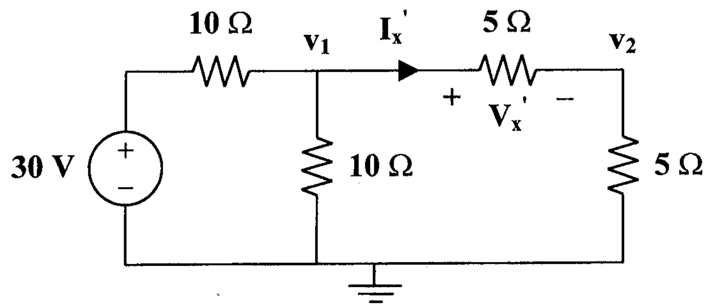
- **PRESENT everything you know about the problem.**
  Using the principle of superposition, we will need to find the desired current when the current source is turned off, or set equal to zero. This implies the replacement of the current source with an open circuit. Let this value be equal to I_x'. We will also need to find the desired current when the voltage source is turned off. This implies the replacement of the voltage source with a short circuit. Let this value be equal to I_x''. Then, the desired current is the sum of these two currents, i.e.,

$$I_x = I_x' + I_x''$$

- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
  This circuit, having two independent sources, can be analyzed using nodal or mesh analysis, as seen in Chapter 3. The problem statement requires the use of superposition. The principle of superposition can be very useful, especially when there are several independent sources. More work may be required to find the desired voltage or current, but the analysis is performed using simpler circuits.

- **ATTEMPT a problem solution.**

Setting the current source to zero, the circuit becomes



Use nodal analysis to find v_1 and v_2.

At node 1 :

$$\frac{v_1-30}{10}+\frac{v_1-0}{10}+\frac{v_1-v_2}{5}=0$$

At node 2 :

$$\frac{v_2-v_1}{5}+\frac{v_2-0}{5}=0$$

Simplifying,

$$v_1-30+v_1+(2)(v_1-v_2)=0$$
$$4v_1-2v_2=30$$
$$2v_1-v_2=15$$

$$v_2-v_1+v_2=0$$
$$-v_1+2v_2=0$$

The system of simultaneous equations is

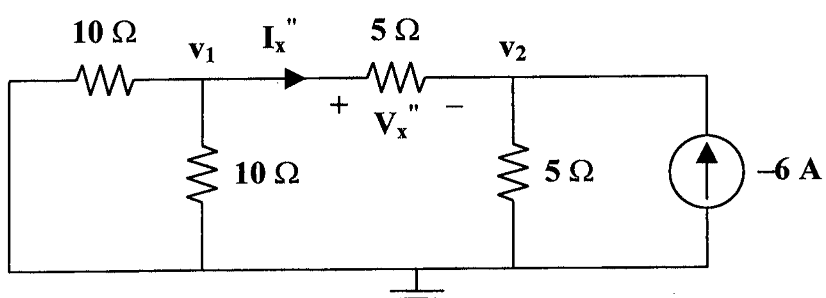
$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} 15 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\frac{1}{4-1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 15 \\ 0 \end{bmatrix}=\frac{1}{3}\begin{bmatrix} 30 \\ 15 \end{bmatrix}=\begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Clearly, $V_x' = v_1 - v_2 = 10 - 5 = 5$ volts.

and $I_x' = \frac{V_x'}{5} = \frac{5}{5} = 1$ amp.

Setting the voltage source to zero, the circuit becomes



Use nodal analysis to find v_1 and v_2.

At node 1 :

$$\frac{v_1 - 0}{10} + \frac{v_1 - 0}{10} + \frac{v_1 - v_2}{5} = 0$$

At node 2 :

$$\frac{v_2 - v_1}{5} + \frac{v_2 - 0}{5} = -6$$

Simplifying,

$$v_1 + v_1 + (2)(v_1 - v_2) = 0$$
$$4v_1 - 2v_2 = 0$$
$$2v_1 - v_2 = 0$$

$$v_2 - v_1 + v_2 = -30$$
$$-v_1 + 2v_2 = -30$$

The system of simultaneous equations is

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{4-1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ -30 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -30 \\ -60 \end{bmatrix} = \begin{bmatrix} -10 \\ -20 \end{bmatrix}$$

Clearly, $V_x'' = v_1 - v_2 = -10 - (-20) = 10\,\text{volts}.$

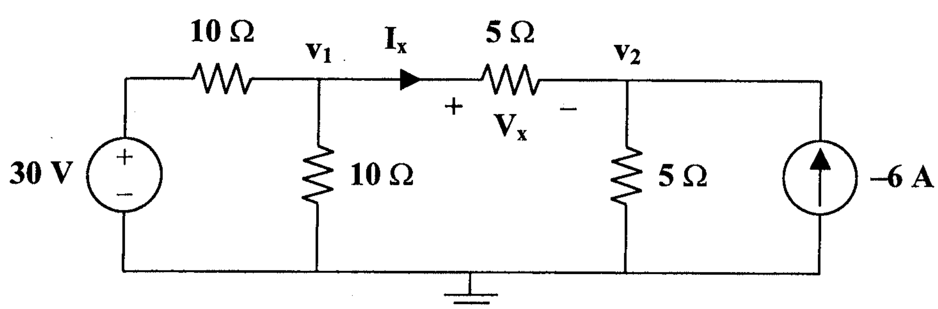
and $I_x'' = \frac{V_x''}{5} = \frac{10}{5} = 2\,\text{amps}.$

The sum of the currents, I_x' and I_x'', due to the two independent sources acting alone is the current, I_x, of the circuit due to both sources.

$$I_x = I_x' + I_x''$$
$$I_x = 1 + 2 = 3\,\text{amps}.$$

- **EVALUATE the solution and check for accuracy.**

Find I_x using both sources and nodal analysis. The circuit is as follows.



At node 1 :

$$\frac{v_1 - 30}{10} + \frac{v_1 - 0}{10} + \frac{v_1 - v_2}{5} = 0$$

At node 2 :

$$\frac{v_2 - v_1}{5} + \frac{v_2 - 0}{5} = -6$$

Simplifying,

$$v_1 - 30 + v_1 + (2)(v_1 - v_2) = 0 \qquad\qquad v_2 - v_1 + v_2 = -30$$
$$4v_1 - 2v_2 = 30 \qquad\qquad -v_1 + 2v_2 = -30$$
$$2v_1 - v_2 = 15$$

The system of simultaneous equations is

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 15 \\ -30 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{4-1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 15 \\ -30 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 30-30 \\ 15-60 \end{bmatrix} = \begin{bmatrix} 0 \\ -15 \end{bmatrix}$$

Clearly, $V_x = v_1 - v_2 = 0 - (-15) = 15$ volts.

and $I_x = \dfrac{V_x}{5} = \dfrac{15}{5} = 3$ amps.

This answer is the same as the answer obtained using the principle of superposition. Our check for accuracy was successful.

- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
  This problem has been solved satisfactorily.

$$I_x = \underline{\mathbf{3\ A}}$$

Problem 4.4 *[4.11]* Apply the superposition principle to find v_o in the circuit of Figure 4.4.

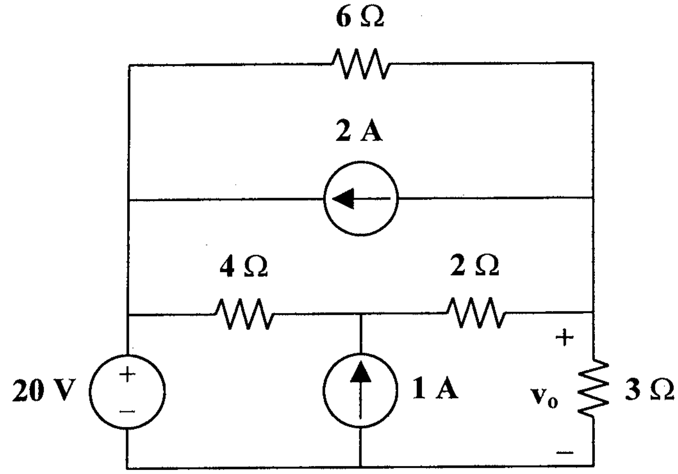
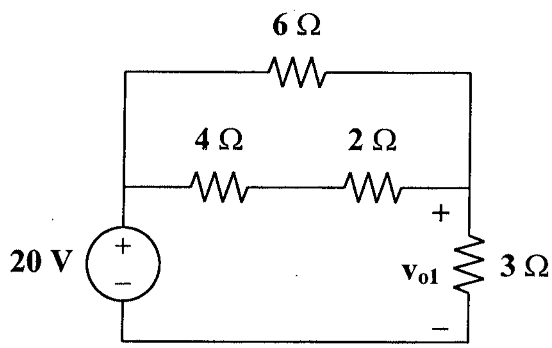


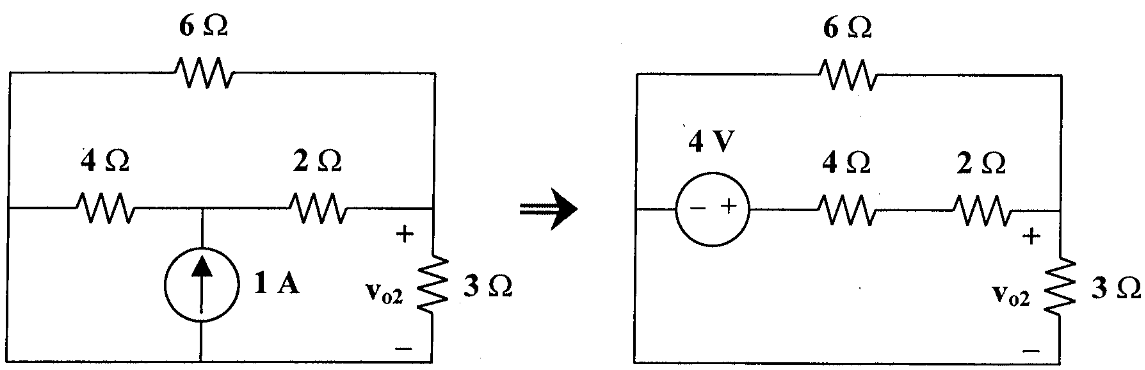
Figure 4.4

Let $v_o = v_{o1} + v_{o2} + v_{o3}$, where v_{o1}, v_{o2}, and v_{o3} are due to the 20-V, 1-A, and 2-A sources, respectively. For v_{o1}, consider the circuit below.



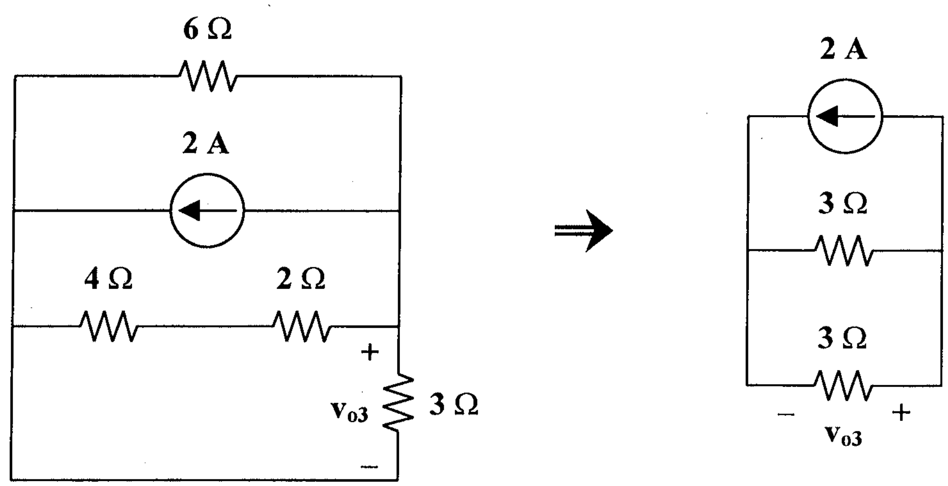
$6 \| (4+2) = 3\,\Omega$ $\qquad\qquad$ $v_{o1} = (1/2)(20) = 10\text{ V}$

For v_{o2}, consider the circuit below.



$3 \| 6 = 2\,\Omega$ $\qquad\qquad$ $v_{o2} = \left(\dfrac{2}{4+2+2}\right)(4) = 1\text{ V}$

For v_{o3}, consider the circuit below.



$$6 \,||\, (4+2) = 3\,\Omega \qquad\qquad v_{o3} = (-1)(3) = -3$$

Thus,

$$v_o = 10 + 1 - 3 = \underline{\mathbf{8\ V}}$$

Problem 4.5 A linear circuit has the following independent sources: V_1, V_2, and I_S along with a current, I_R, through a resistor. It cannot be assumed that these are the only elements (resistors and dependent sources) in the circuit. Complete the following table.

Circuit number	V_1 (volts)	V_2 (volts)	I_S (amps)	I_R (amps)
Condition #1	10	0	0	2
Condition #2	0	5	0	–1
Condition #3	0	0	3	1
Condition #4	10	5	3	
Condition #5	10	–20	3	
Condition #6				6

This problem is based on linearity and the principle of superposition.

From the Conditions #1, #2, and #3, we know

- $I_R = 2$ amps when the only contributing independent source is $V_1 = 10$ volts.
- $I_R = -1$ amp when the only contributing independent source is $V_2 = 5$ volts.
- $I_R = 1$ amp when the only contributing independent source is $I_S = 3$ amps.

Condition #4 :

$$I_R = \left(\frac{10}{10}\right)(2) + \left(\frac{5}{5}\right)(-1) + \left(\frac{3}{3}\right)(1) = 2 - 1 + 1 = \underline{\mathbf{2\ A}}$$

Condition #5 :

$$I_R = \left(\frac{10}{10}\right)(2) + \left(\frac{-20}{5}\right)(-1) + \left(\frac{3}{3}\right)(1) = 2 + 4 + 1 = \underline{\mathbf{7\ A}}$$

Condition #6 : The following are only three of an infinite set of solutions.

If only one source is not equal to zero

then $V_1 = ?$ $\quad V_2 = \underline{\mathbf{0}}$ $\quad I_S = \underline{\mathbf{0}}$ $\quad I_R = 6$ $\qquad 6 = \left(\frac{V_1}{10}\right)(2)$ $\qquad V_1 = \underline{\mathbf{30}}$

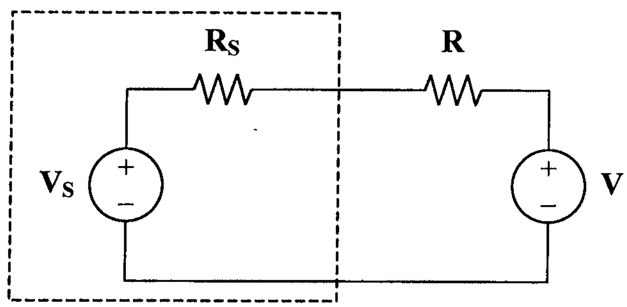
or $V_1 = \underline{\mathbf{0}}$ $\quad V_2 = ?$ $\quad I_S = \underline{\mathbf{0}}$ $\quad I_R = 6$ $\qquad 6 = \left(\frac{V_2}{5}\right)(-1)$ $\qquad V_2 = \underline{\mathbf{-30}}$

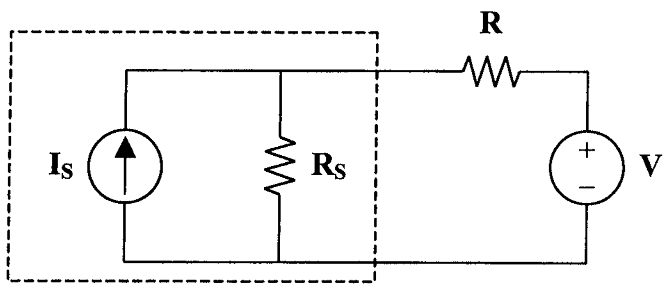
or $V_1 = \underline{\mathbf{0}}$ $\quad V_2 = \underline{\mathbf{0}}$ $\quad I_S = ?$ $\quad I_R = 6$ $\qquad 6 = \left(\frac{I_S}{3}\right)(1)$ $\qquad I_S = \underline{\mathbf{18}}$

SOURCE TRANSFORMATIONS

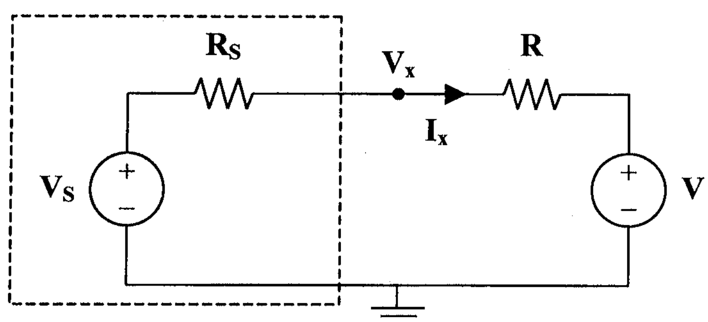
A source transformation is the process of replacing a voltage source V_S in series with a resistor R with a current source I_S in parallel with a resistor R or vice versa.

Problem 4.6 Given that the following circuits are linear, prove that the two circuits are equivalent, where $R_S = V_S / I_S$.





Two circuits are said to be equivalent if they have the same voltage-current relationship at their terminals. Begin by finding V_x and I_x in the following circuit.



Clearly,

$$I_x = \frac{V_S - V_x}{R_S}$$

or

$$V_x = V_S - R_S I_x$$

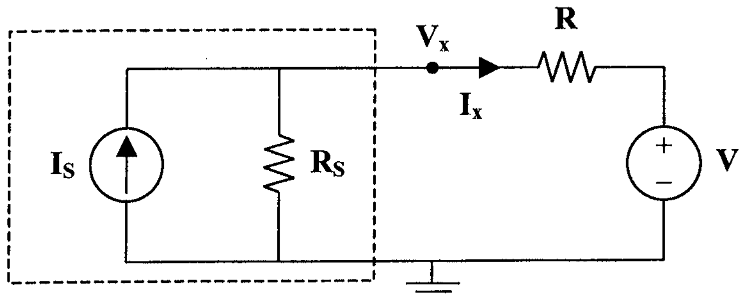
Using KVL,

$$-V_S + (R_S + R)I_x + V = 0$$

or

$$I_x = \frac{V_S - V}{R_S + R}$$

Now, find V_x and I_x in this circuit.



Using KCL,

$$I_S = I_x + \frac{V_x}{R_S}$$

or

$$V_x = R_S I_S - R_S I_x$$

We know that $R_S = \frac{V_S}{I_S}$. So,

$$V_x = V_S - R_S I_x.$$

Using mesh analysis,

$$R I_x + V + R_S (I_x - I_S)$$
$$(R_S + R) I_x = R_S I_S - V$$
$$I_x = \frac{R_S I_S - V}{R_S + R}$$

We know that $R_S = \frac{V_S}{I_S}$. So,

$$I_x = \frac{V_S - V}{R_S + R}.$$

In both circuits, $V_x = V_S - R_S I_x$ and $I_x = \frac{V_S - V}{R_S + R}$.

Therefore, **<u>the two circuits are equivalent</u>**.

Problem 4.7 *[4.23]* Given the circuit in Figure 4.5, use source transformation to find i_o.

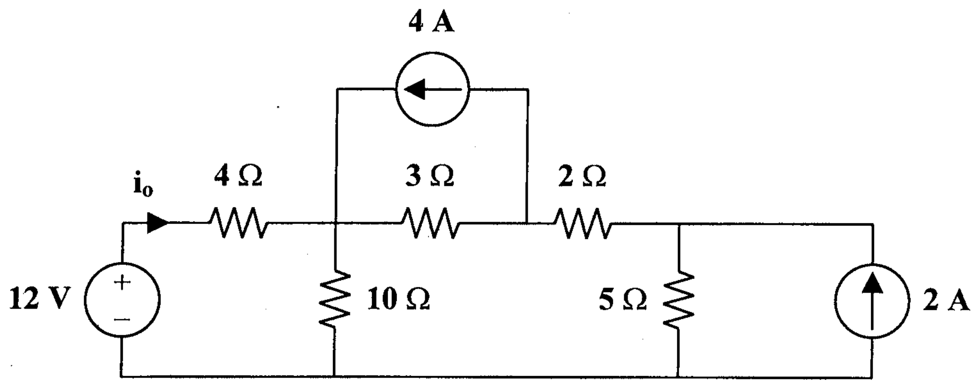
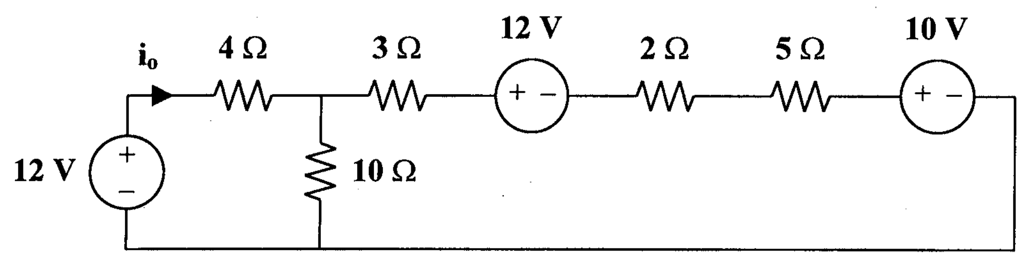
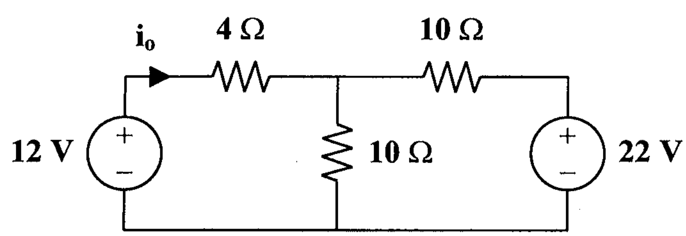


Figure 4.5

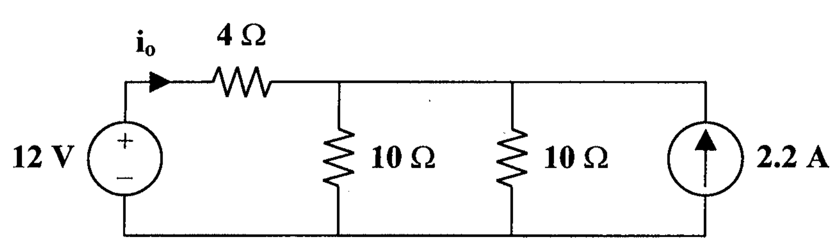
Transforming only the current sources leads to Fig. (a). Continuing with source transformations finally produces the circuit in Fig. (d).



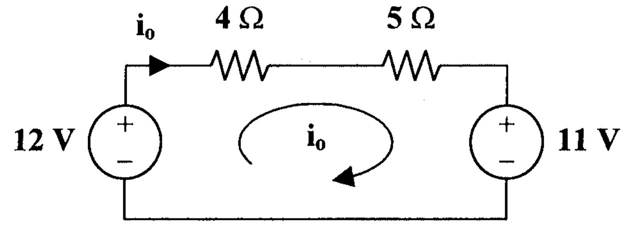
(a)



(b)



(c)



(d)

Applying KVL to the loop in Fig. (d),

$$-12 + 9i_o + 11 = 0 \longrightarrow i_o = 1/9 = \underline{\mathbf{111.11\ mA}}$$

Problem 4.8 Using source transformations, solve for I_o in Figure 4.6.

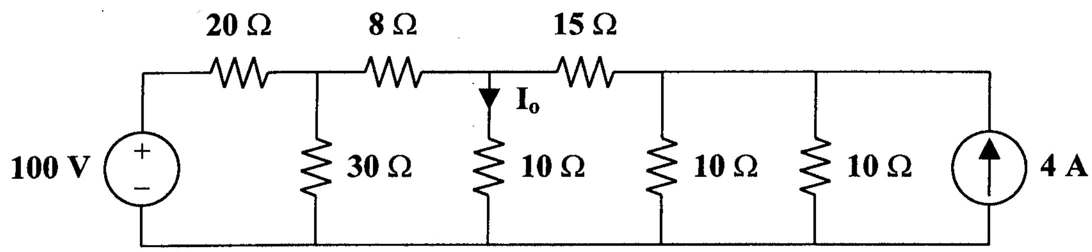
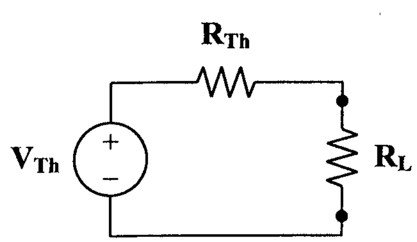


Figure 4.6

$$I_o = \underline{\mathbf{2\ A}}$$

THEVENIN'S THEOREM

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are zero. An alternative way to find R_{Th} is to find the short-circuit current between the terminals, I_{sc}; then,

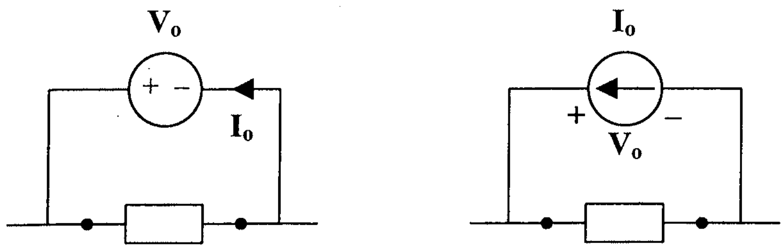


$$V_{Th} = V_{oc}$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}}$$

Note that finding the equivalent resistance between the terminals using this approach is a valid method only if there are no dependent sources. Calculating R_{Th} using V_{oc} and I_{sc} is a valid method with independent and dependent sources.

Finally, if $V_{oc} = I_{sc} = 0$, then the circuit must be excited with a voltage source or a current source at the terminals, as shown below, to find R_{Th}.



Find I_o if given a voltage source $V_o = 1$ V. Find V_o if given a current source $I_o = 1$ A. Now, $R_{Th} = 1/I_o$ or $R_{Th} = V_o/1$.

Problem 4.9 Solve for the Thevenin equivalent circuit as seen by the 10-Ω resistor with the current I_o flowing through it in Figure 4.6.

➢ **Carefully DEFINE the problem.**
Each component is labeled completely. The problem is clear.

➢ **PRESENT everything you know about the problem.**
To find the Thevenin equivalent circuit, we need to find the open-circuit voltage (V_{oc}) across the terminals of the 10-Ω resistor with the current I_o flowing through it. We also need to find the short-circuit current (I_{sc}) through these terminals. Then,

$$V_{Th} = V_{oc} \qquad \text{and} \qquad R_{Th} = V_{oc} / I_{sc}$$

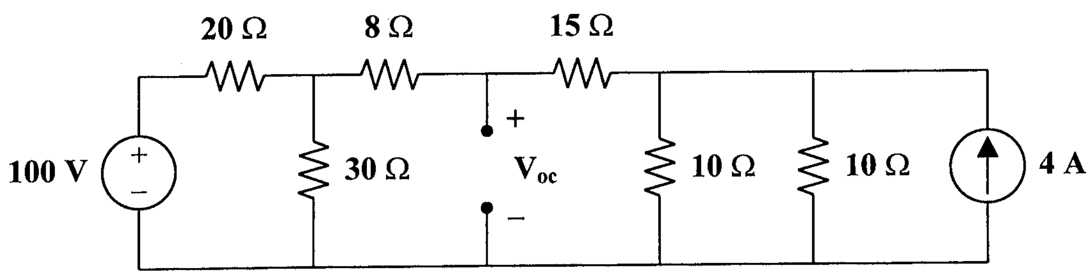
Because the circuit has no dependent sources, there is an alternative way to find the Thevenin resistance. R_{Th} is the input resistance at the terminals when the independent sources are set equal to zero. This method will not be used in the initial attempt to find a solution. It is better to find the short-circuit current and calculate the Thevenin resistance because it will work for any circuit. However, we can find the input resistance at the terminals to check our initial solution.

➢ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
To find the open-circuit voltage, the obvious techniques are nodal analysis, mesh analysis, or source transformations along with KVL. Because we want to find a voltage, let's eliminate mesh analysis as a promising technique. It is clear that nodal analysis produces a set of three equations and three unknowns. Simplifying the circuit using source transformations yields a single loop from which we can easily find the open-circuit voltage.

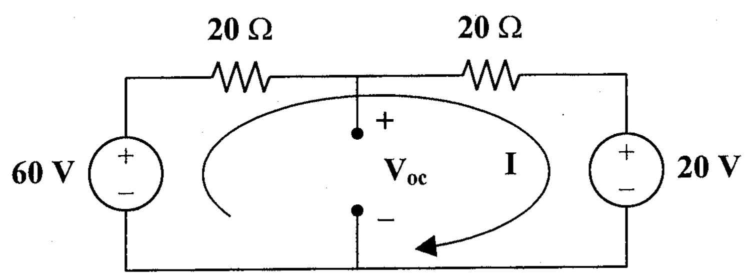
To find the short-circuit current, a similar argument can be made. So, let's eliminate nodal analysis because we are looking for a current rather than a voltage. Mesh analysis gives a set of equations to be solved. Lastly, source transformations allows the circuit to be reduced to a single loop from which we can easily find the short-circuit current.

With the open-circuit voltage and the short-circuit current at the terminals, the Thevenin equivalent circuit is found as stated above.

➤ **ATTEMPT a problem solution.**
Begin by finding the open-circuit voltage as seen by the element.



After a couple of source transformations, as seen in Problem 4.8, the circuit becomes



Using KVL, find V_{oc}.

$$-60 + 20I + 20I + 20 = 0$$

$$40I = 40 \longrightarrow I = 1 \text{ amp}$$

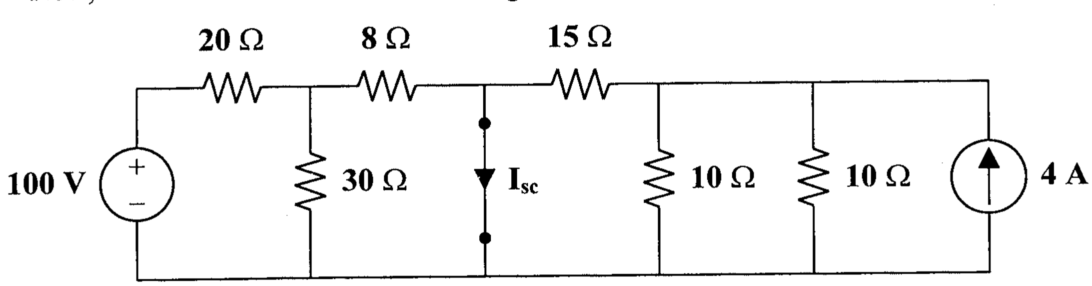
where

$$I = \frac{60 - V_{oc}}{20} = \frac{V_{oc} - 20}{20} = 1$$

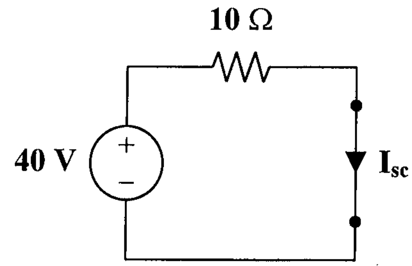
$$60 - V_{oc} = V_{oc} - 20$$

$$V_{oc} = 40 \text{ volts}$$

Now, find the short-circuit current through the element.



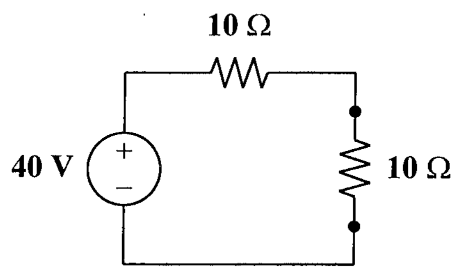
Again using source transformations, the circuit can be reduced as shown below.



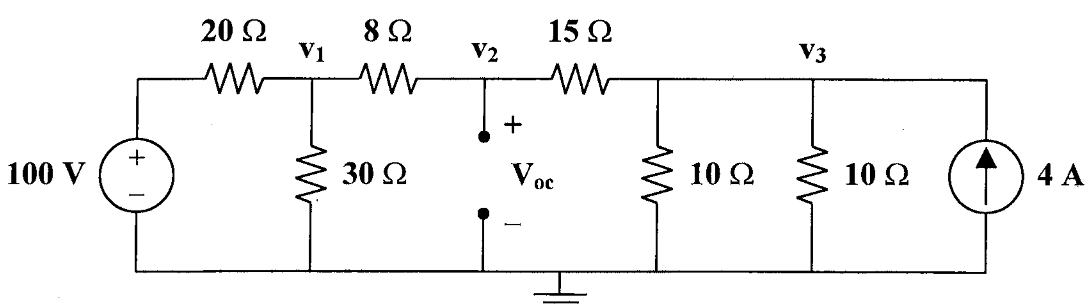
Clearly, $$I_{sc} = \frac{40}{10} = 4 \text{ amps.}$$

Then, $$R_{Th} = \frac{40}{10} = 4 \text{ ohms.}$$

Therefore, the Thevenin equivalent circuit is



- **EVALUATE the solution and check for accuracy.**
  To check the open-circuit voltage, perform nodal analysis using the circuit below.



For node 1, $$\frac{v_1 - 100}{20} + \frac{v_1}{30} + \frac{v_1 - v_2}{8} = 0$$

For node 2, $$\frac{v_2 - v_1}{8} + \frac{v_2 - v_3}{15} = 0$$

For node 3, $$\frac{v_3 - v_2}{15} + \frac{v_3}{10} + \frac{v_3}{10} - 4 = 0$$

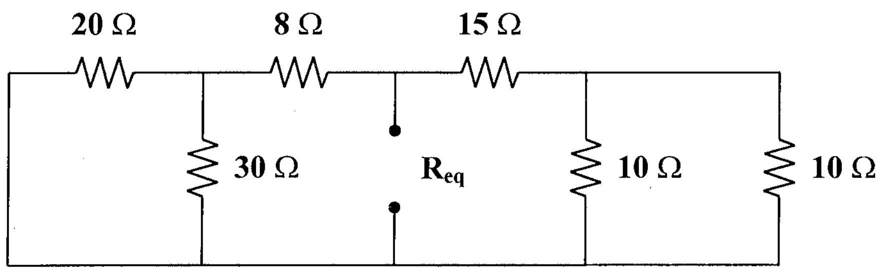
Simplifying these equations and putting them into matrix form results in

$$\begin{bmatrix} 50 & -30 & 0 \\ -15 & 23 & -8 \\ 0 & -2 & 8 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1200 \\ 0 \\ 120 \end{bmatrix}.$$

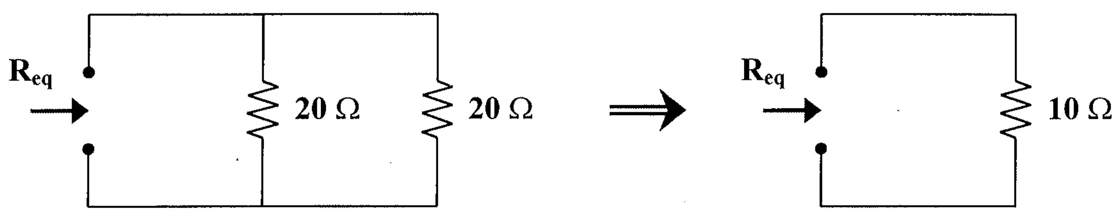
which yields $\quad v_1 = 48\,\text{volts} \quad\quad v_2 = 40\,\text{volts} \quad\quad v_3 = 25\,\text{volts}$

Hence, $\quad V_{Th} = V_{oc} = v_2 = 40\,\text{volts}$

Since there are only independent sources, an alternative way to find R_{Th} is to set the sources to zero and calculate the resistance of the modified network. After replacing the voltage source with a short circuit and the current source with an open circuit, the network becomes



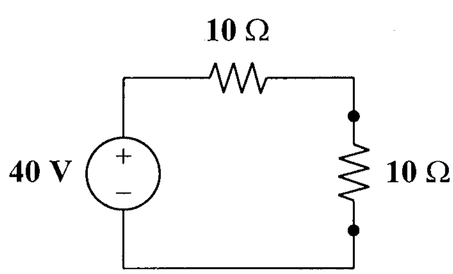
Combine the resistors to the left of the open circuit. $(20\ \Omega \,||\, 30\ \Omega) + 8\ \Omega = 20\ \Omega$
Combine the resistors to the right of the open circuit. $(10\ \Omega \,||\, 10\ \Omega) + 15\ \Omega = 20\ \Omega$



An equivalent resistance of 10 Ω matches the value that was calculated using V_{oc} and I_{sc}.

Our check for accuracy was successful.

- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
  This problem has been solved satisfactorily. **<u>The Thevenin equivalent circuit is as follows</u>**.



Problem 4.10 Find the Thevenin equivalent as seen by R in Figure 4.7.

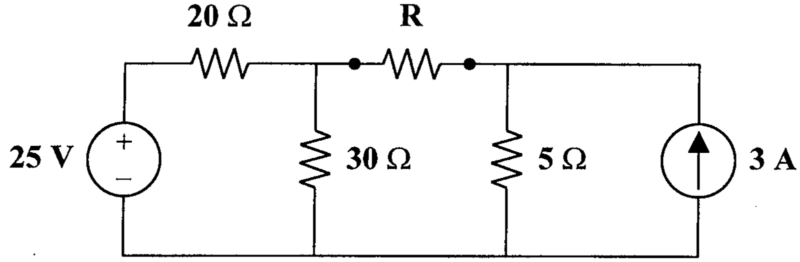
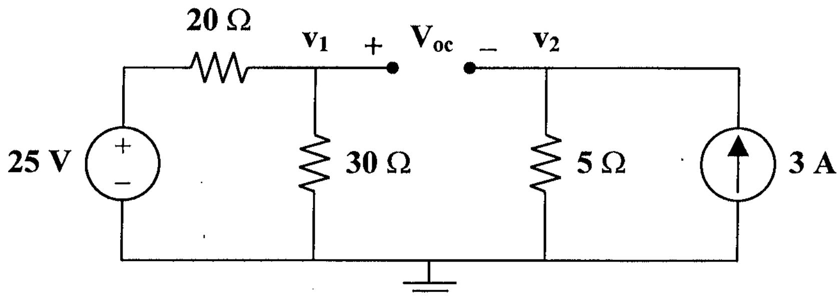


Figure 4.7

First, find the open-circuit voltage, where $V_{Th} = V_{oc}$.



Using nodal analysis,
At node 1 :

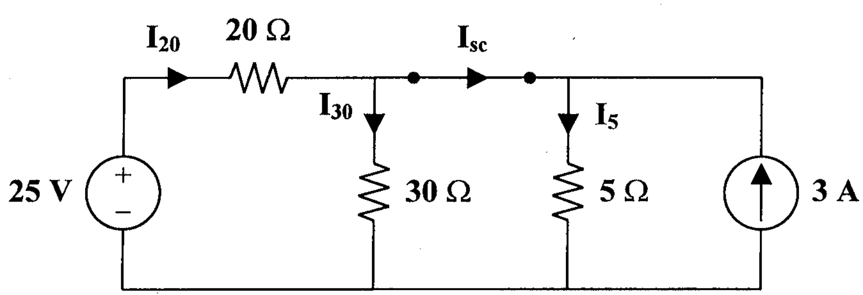
$$\frac{v_1 - 25}{20} + \frac{v_1 - 0}{30} = 0$$
$$3v_1 + 2v_1 = 75$$
$$v_1 = 15$$

At node 2 :

$$\frac{v_2 - 0}{5} = 3$$
$$v_2 = 15$$

Clearly, $V_{oc} = v_1 - v_2 = 15 - 15 = 0$ volts.

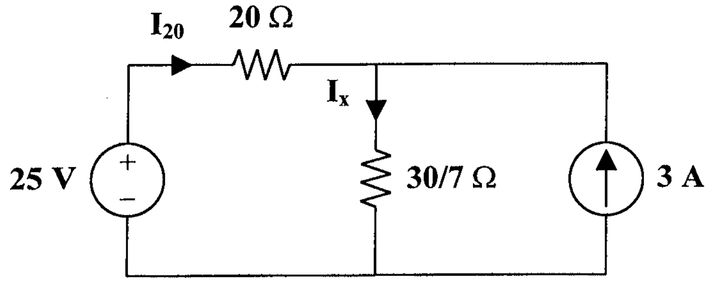
Now, find the short-circuit current.



Clearly, $I_{sc} = I_{20} - I_{30}$.
Perform KVL using the left loop.

$$-25 + 20I_{20} + 30I_{30} = 0$$

Begin by finding I_{20} in the following modified circuit.



$$-25 + 20I_{20} + \frac{30}{7}I_x = 0 \qquad \text{where } I_x = I_{20} + 3$$
$$140I_{20} + (30)(I_{20} + 3) = 175$$
$$170I_{20} = 85$$
$$I_{20} = 0.5 \text{ amps}$$

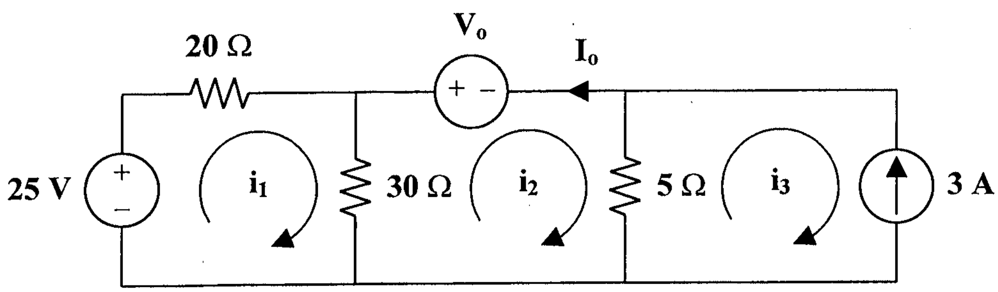
Now,

$$-25 + (20)(0.5) + 30I_{30} = 0$$
$$I_{30} = 15/30 = 0.5 \text{ amps}$$

Finally,

$$I_{sc} = I_{20} - I_{30} = 0.5 - 0.5 = 0 \text{ amps.}$$

Since 0/0 is undefined, we need to excite the circuit with a 1-V voltage source at the terminals of R in order to find R_{Th}.



Use mesh analysis to find I_o. Then, $R_{Th} = V_o / I_o$.

For loop 1 : $-25 + 20i_1 + (30)(i_1 - i_2) = 0$

For loop 2 : $(30)(i_2 - i_1) + V_o + (5)(i_2 - i_3) = 0$ where $V_o = 1$ volt

For loop 3 : $i_3 = -3$ This is the constraint equation.

Simplifying,

$$50i_1 - 30i_2 = 25$$
$$-30i_1 + 35i_2 = -16$$

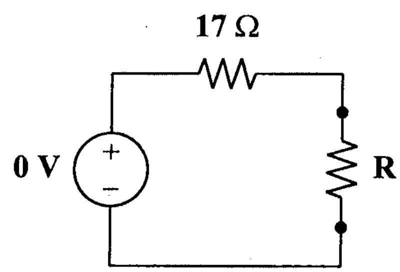
In matrix form,

$$\begin{bmatrix} 50 & -30 \\ -30 & 35 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 25 \\ -16 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0.4647 \\ -0.0588 \end{bmatrix}$$

Now, $I_o = -i_2 = 0.0588$ and $R_{Th} = \frac{V_o}{I_o} = \frac{1}{0.0588} = 17$ ohms.

Therefore, **<u>the Thevenin equivalent circuit is as follows</u>**.



Problem 4.11 *[4.33]* For the circuit in Figure 4.8, find the Thevenin equivalent between terminals a and b.

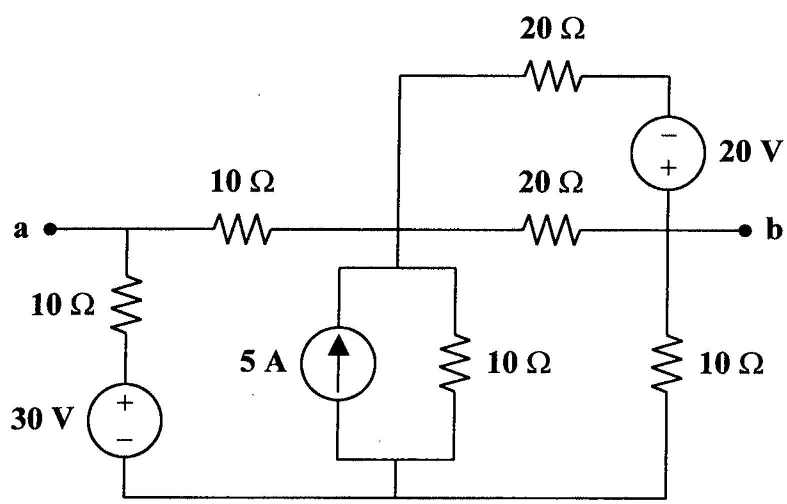
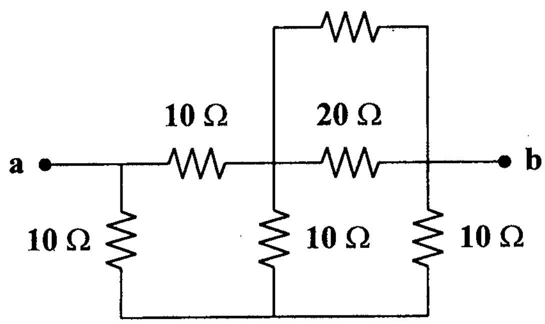


Figure 4.8

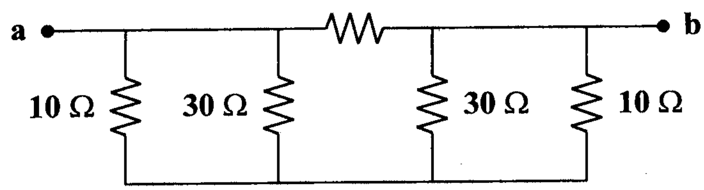
To find R_{Th}, consider the circuit in Fig. (a).



(a)

where $20 \,||\, 20 = 10\,\Omega$.

Now, transform the wye subnetwork to a delta as shown in Fig. (b).

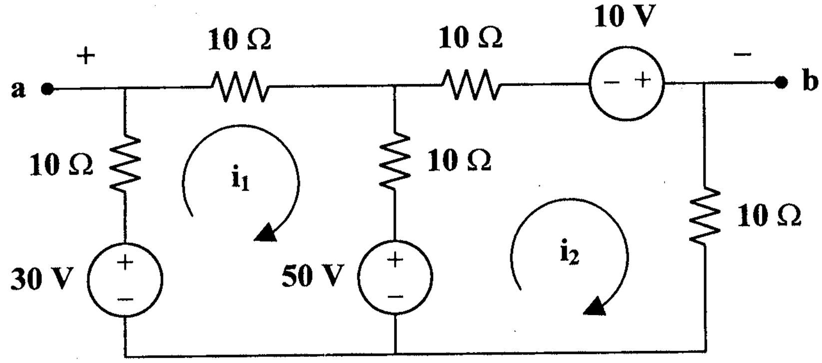


(b)

where $10 \,||\, 30 = 7.5\,\Omega$.

Hence, $$R_{Th} = R_{ab} = 30 \,||\, (7.5 + 7.5) = 30 \,||\, 15 = \underline{\mathbf{10\,\Omega}}$$

To find V_{Th}, we transform the 20-V and the 5-V sources to obtain the circuit shown in Fig. (c).



(c)

For loop 1,

$$-30 + 50 + 30i_1 - 10i_2 = 0$$

$$-2 = 3i_1 - i_2 \qquad (1)$$

For loop 2,

$$-50 - 10 + 30i_2 - 10i_1 = 0$$

$$6 = -i_1 + 3i_2 \qquad (2)$$

Solving (1) and (2),

$i_1 = 0\ \text{A}$ and $i_2 = 2\ \text{A}$

Applying KVL to the output loop,

$$-V_{ab} - 10i_1 + 30 - 10i_2 = 0$$

$$V_{ab} = 10\ \text{V}$$

$$V_{Th} = V_{ab} = \underline{\mathbf{10\ V}}$$

Problem 4.12 Find the Thevenin equivalent as seen by R in Figure 4.9.

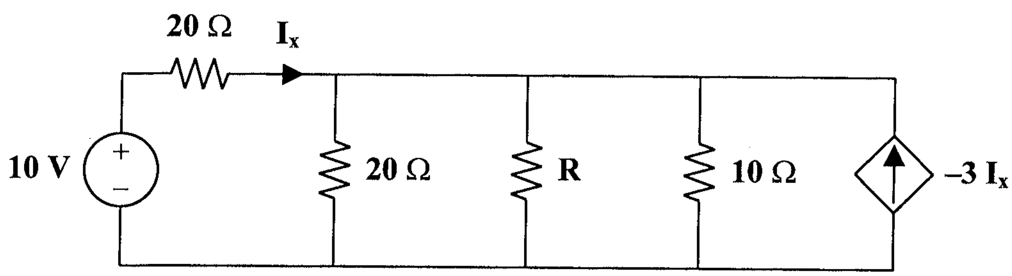
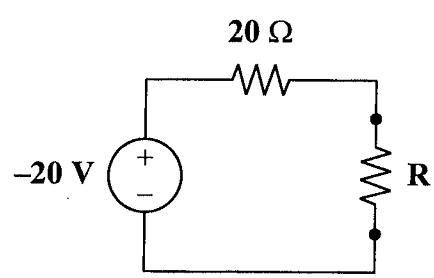


Figure 4.9

Therefore, **<u>the Thevenin equivalent circuit is as follows</u>**.



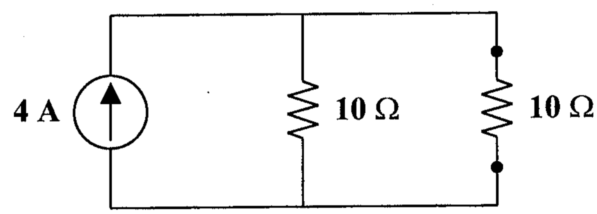
NORTON'S THEOREM

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current at the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Problem 4.13 Find the Norton equivalent of the circuit in Figure 4.6.

In Problem 4.9, $V_{oc} = 40$ volts, $I_{sc} = 4$ amps, and $R_{Th} = 10$ ohms.
Recall that $R_N = R_{Th}$; so $R_N = 10$ ohms.

Therefore, **the Norton equivalent circuit is as shown**.



Problem 4.14 *[4.41]* Given the circuit in Figure 4.10, obtain the Norton equivalent as viewed from terminals:
(a) a-b (b) c-d

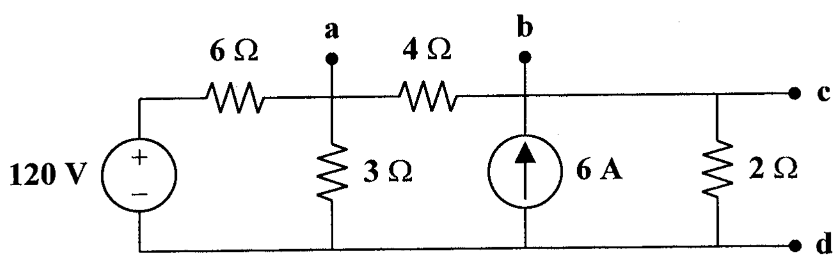
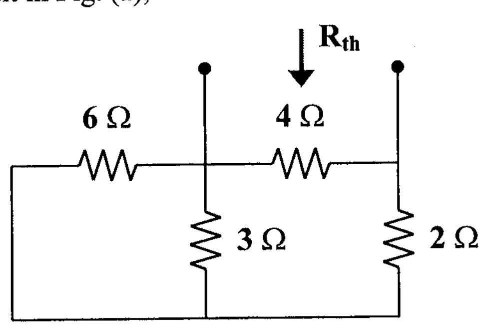


Figure 4.10

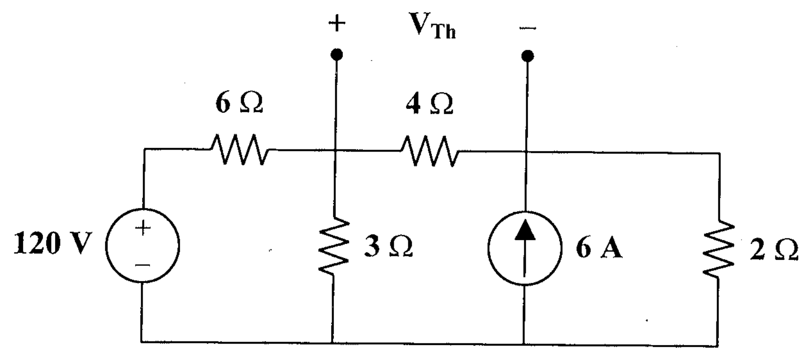
(a) From the circuit in Fig. (a),



(a)

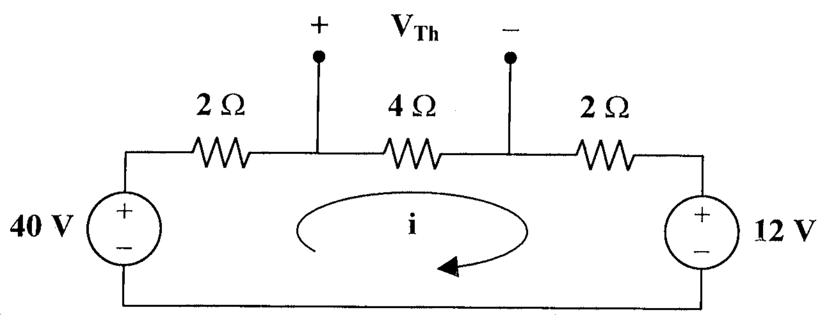
$$R_N = 4 \| (2 + 6 \| 3) = 4 \| 4 = \underline{2\ \Omega}$$

For I_N or V_{Th}, consider the circuit in Fig. (b).



(b)

After some source transformations, the circuit becomes that shown in Fig. (c).



(c)

Applying KVL to the circuit in Fig. (c),

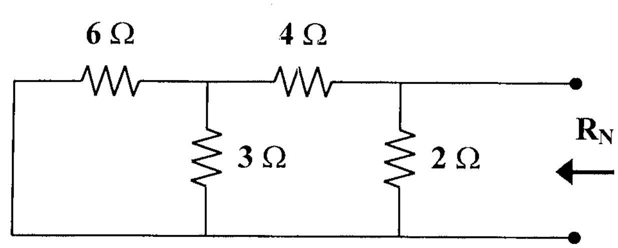
$$-40 + 8i + 12 = 0 \longrightarrow i = 7/2$$

Hence,

$$V_{Th} = 4i = 14\ V$$

and $\quad I_N = V_{Th}/R_N = 14/2 = \underline{7\ A}$

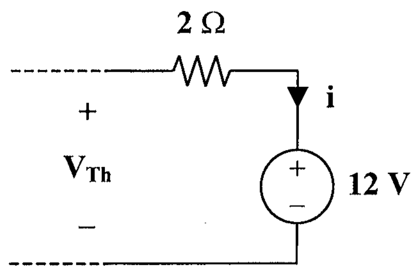
(b) To get R_N, consider the circuit in Fig. (d).



(d)

$$R_N = 2 \,||\, (4 + (6 \,||\, 3) = 2 \,||\, 6 = \underline{\mathbf{1.5\ \Omega}}$$

To get I_N, the circuit in Fig. (c) applies except that it needs slight modification as in Fig. (e).



(e)

$$i = (40 - 12)/(2 + 4 + 2)\, 7/2 \quad \text{and} \quad V_{Th} = 12 + 2i = 19$$

$$I_N = V_{Th}/R_N = 19/1.5 = \underline{\mathbf{12.667\ A}}$$

Problem 4.15 Find the Norton equivalent of the circuit in Figure 4.11.

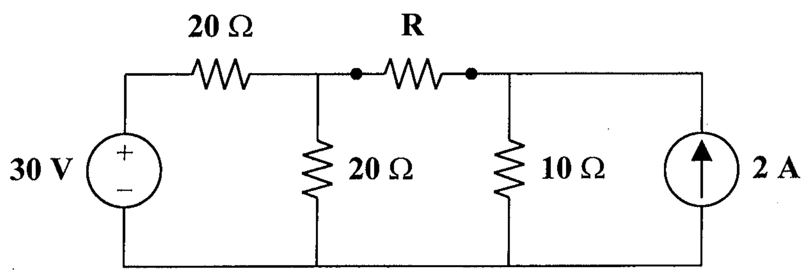
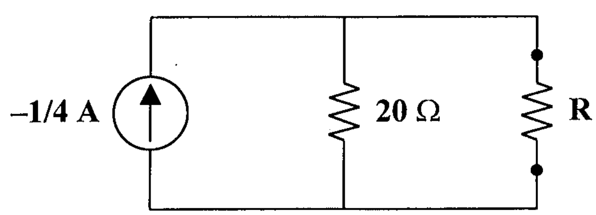


Figure 4.11

Therefore, **<u>the Norton equivalent circuit is as follows</u>**.



MAXIMUM POWER TRANSFER

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen by the load ($R_L = R_{Th}$).

Problem 4.16 Given the circuit in Figure 4.12, complete the following table.

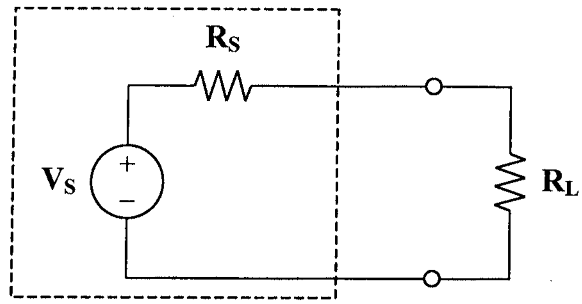


Figure 4.12

	V_S (volts)	R_S (ohms)	R_L (ohms)	Power of R_L (watts)
Condition #1	20	10		
Condition #2	V	10		
Condition #3	20		10	
Condition #4	20			

Let p be the power of the load resistor, R_L. Then, $p = \left(\dfrac{V_S}{R_S + R_L} \right)^2 R_L$.

We want to find the maximum power transferred to the load. The maximum power is transferred to the load when the load resistance is equivalent to the Thevenin resistance as seen by the load.

In this case, $R_L = R_S$ and $p = \dfrac{V_S^2}{4R_L}$

Condition #1 : $R_L = R_S = \underline{\mathbf{10\,\Omega}}$ $\qquad p = \dfrac{(20)^2}{(4)(10)} = \dfrac{400}{40} = \underline{\mathbf{10\ watts}}$

Condition #2 : $R_L = R_S = \underline{\mathbf{10\,\Omega}}$ $\qquad p = \dfrac{V^2}{(4)(10)} = \underline{\dfrac{\mathbf{V^2}}{\mathbf{40}}\ \mathbf{watts}}$

Condition #3 : $p = \left(\dfrac{20}{R_S + 10} \right)^2 (10)$

Clearly, for maximum power, $R_S = \underline{\mathbf{0\,\Omega}}$. *{with R_L fixed, making R_s as small as possible maximizes the power to R_L}*

Then, $p = \left(\dfrac{20}{10} \right)^2 (10) = (2^2)(10) = \underline{\mathbf{40\ watts}}$.

Condition #4 : For maximum power, $R_S = \underline{\mathbf{0\,\Omega}}$ and $R_L = \underline{\mathbf{0^+}\ \Omega}$.

Then, $p = \dfrac{20^2}{(4)(0^+)} = \underline{\infty^- \textbf{ watts}}$.

Problem 4.17 *[4.59]*

(a) For the circuit in Figure 4.13, obtain the Thevenin equivalent at terminals a-b.

(b) Calculate the current in $R_L = 8\,\Omega$.

(c) Find R_L for maximum power deliverable to R_L.

(d) Determine that maximum power.

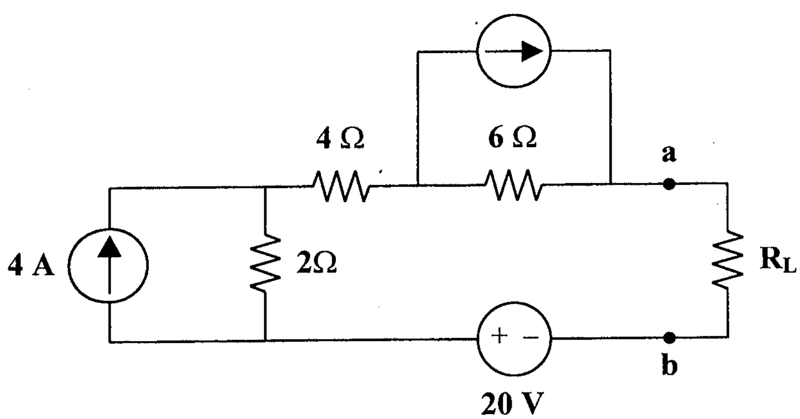
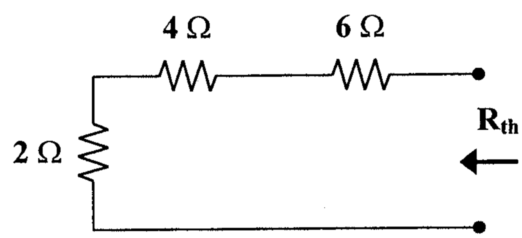


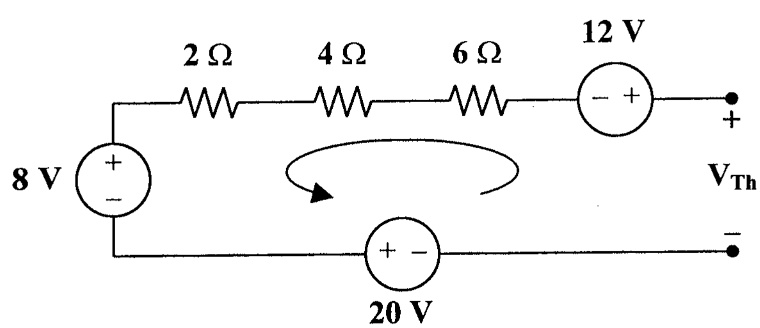
Figure 4.13

(a) R_{Th} and V_{Th} are calculated using the circuits in Fig. (a) and (b) respectively.



(a)

From Fig. (a), $R_{Th} = 2 + 4 + 6 = \underline{12\,\Omega}$



(b)

From Fig. (b),

$$-V_{Th} + 12 + 8 + 20 = 0$$
$$V_{Th} = \underline{\mathbf{40\ V}}$$

(b) $$i = \frac{V_{Th}}{R_{Th} + R} = \frac{40}{12 + 8} = \underline{\mathbf{2\ A}}$$

(c) For maximum power transfer,
$$R_L = R_{Th} = \underline{\mathbf{12\ \Omega}}$$

(d) $$p = \frac{V_{Th}^{\ 2}}{4 R_{Th}} = \frac{(40)^2}{(4)(12)} = \underline{\mathbf{33.33\ watts}}$$

Problem 4.18 Using Figure 4.14, what values of R_S and R_L result in maximum power delivered to the load? What is the power absorbed by the load?

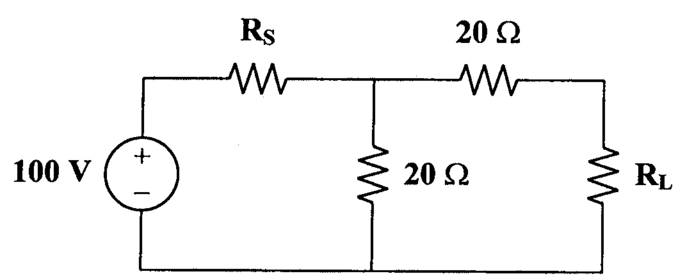


Figure 4.14

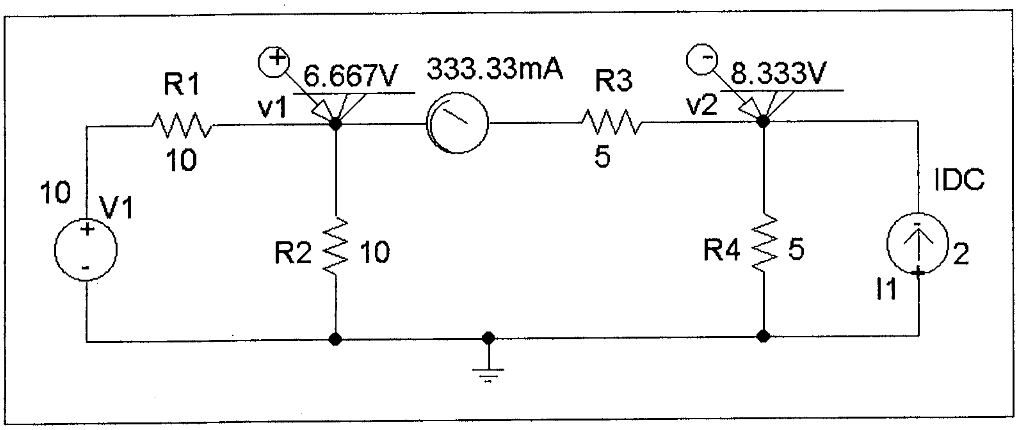
$$R_S = \underline{\mathbf{0\ \Omega}}$$

$$R_L = \underline{\mathbf{20\ \Omega}}$$

$$p_{RL} = \underline{\mathbf{125\ watts}}$$

VERIFYING CIRCUIT THEOREMS WITH PSPICE

Problem 4.19 Solve Problem 4.1 using PSpice.



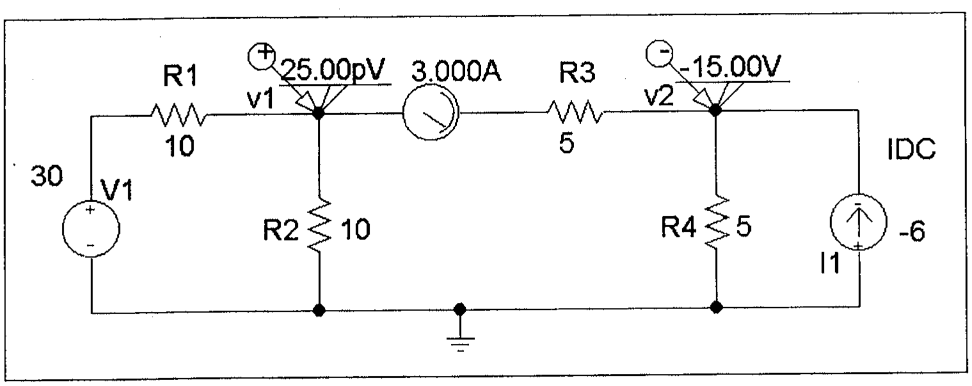
A voltage source in PSpice needs a value to simulate rather than performing a symbolic simulation. So, let $K = 1$. Then, $V_x = -5/3$ volts and $I_x = -1/3$ amps. Clearly, V_x is the voltage across R3 and I_x is the current flowing through R3 from left to right. Thus, for $\underline{\mathbf{K = 1}}$,

$$V_x = v_1 - v_2 = 6.667 - 8.333 = -1.667 = \underline{\mathbf{-5/3\ V}}$$

and $\quad I_x = V_x/5 = -1/3$ amps or $I_x = \underline{\mathbf{-333.333\ mA}}$.

These answers agree with the answers obtained in Problem 4.1 given that $K = 1$.

Problem 4.20 Solve Problem 4.3 using PSpice.



Clearly, V_x is the voltage across R3 and I_x is the current flowing through R3 from left to right. Thus, $V_x = v_1 - v_2 = (25 \times 10^{-12}) - (-15) = 15$ V and $I_x = V_x/5 = \underline{\mathbf{3\ A}}$.

This answer agrees with the answer obtained in Problem 4.3.

CHAPTER 6 - CAPACITORS AND INDUCTORS

List of topics for this chapter :
- Capacitors
- Series and Parallel Capacitors
- Inductors
- Series and Parallel Inductors
- Applications

CAPACITORS

Problem 6.1 For the circuit shown in Figure 6.1, find $i_c(t)$ given

(a) $v(t) = (2t + 6)$ V

(b) $v(t) = 2\cos(\omega t)$ V

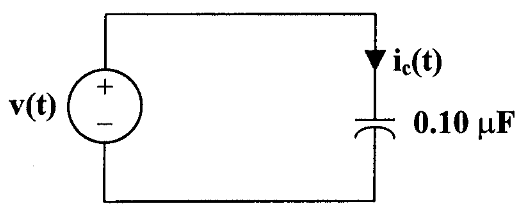


Figure 6.1

(a) $$i_c(t) = C\frac{dv_c}{dt} = 10^{-7}\,\frac{d(2t+6)}{dt} = 2\times 10^{-7} = \underline{\mathbf{0.2\ \mu A}}$$

(b) $$i_c(t) = C\frac{dv_c}{dt} = 10^{-7}\,\frac{d(2\cos(\omega t))}{dt} = (10^{-7})(-2\omega)\sin(\omega t)$$
$$i_c(t) = \underline{\mathbf{-\,0.2\omega\sin(\omega t)\ \mu A}}$$

Problem 6.2 Find $v_c(t)$ as shown in Figure 6.2, given that

(a) $$i(t) = \begin{cases} 0\text{ A} & t < 0 \\ 1\text{ A} & 0 < t < 5 \\ 0\text{ A} & 5 < t \end{cases}$$

(b) $$i(t) = \begin{cases} 0\text{ A} & t < 0 \\ t^2\text{ A} & 0 < t < 2 \\ 0\text{ A} & 2 < t \end{cases}$$

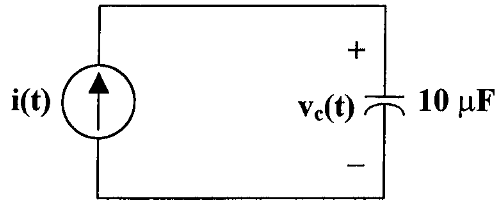


Figure 6.2

(a) $$v_c(t) = \frac{1}{C} \int i_c(t)\, dt$$

For $t < 0$, $v_c(t) = 0\ \text{V}$

For $0 < t < 5$, $v_c(t) = \frac{1}{10^{-5}} \int_0^t 1\, dt = \frac{t}{10^{-5}} = t \times 10^5\ \text{V}$

For $5 < t$, $v_c(t) = 5 \times 10^5\ \text{V}$

$$v_c(t) = \begin{cases} \mathbf{0\ V} & \mathbf{t < 0} \\ \mathbf{t \times 10^5\ V} & \mathbf{0 < t < 5} \\ \mathbf{5 \times 10^5\ V} & \mathbf{5 < t} \end{cases}$$

These voltages are quite large. This is due to the large currents and small capacitances. Normally, the currents would be quite small, in the μA range.

(b) $$v_c(t) = \frac{1}{C} \int i_c(t)\, dt$$

For $t < 0$, $v_c(t) = 0\ \text{V}$

For $0 < t < 2$, $v_c(t) = \frac{1}{10^{-5}} \int_0^t t^2\, dt = \frac{t^3}{3} \times 10^5\ \text{V}$

For $2 < t$, $v_c(t) = \frac{8}{3} \times 10^5\ \text{V}$

$$v_c(t) = \begin{cases} \mathbf{0\ V} & \mathbf{t < 0} \\ \mathbf{\frac{t^3}{3} \times 10^5\ V} & \mathbf{0 < t < 2} \\ \mathbf{2.667 \times 10^5\ V} & \mathbf{2 < t} \end{cases}$$

Problem 6.3 *[6.11]* A voltage of $60\cos(4\pi t)$ V appears across the terminals of a 3-mF capacitor. Calculate the current through the capacitor and the energy stored in it from $t = 0$ to $t = 0.125$ s.

$$i = C\frac{dv}{dt} = (3\times10^{-3})\frac{d(60\cos(4\pi t))}{dt}$$

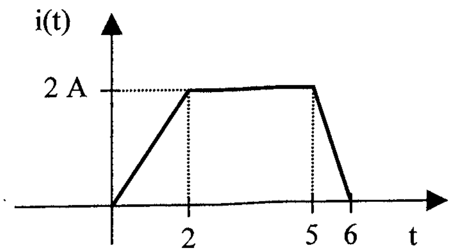
$$i = (3\times10^{-3})(60)(4\pi)[-\sin(4\pi t)] = \underline{\mathbf{-\,0.72\pi\sin(4\pi t)\ A}}$$

$$p = vi = [60\cos(4\pi t)][-0.72\pi\sin(4\pi t))] = -\,21.6\pi\sin(8\pi t)\ W$$

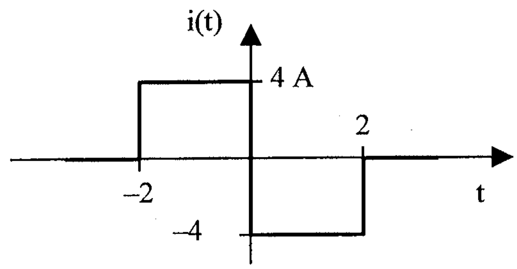
$$w = \int_0^t p\,dt = -21.6\pi\int_0^{1/8}\sin(8\pi t)\,dt$$

$$w = \frac{21.6\pi}{8\pi}\cos(8\pi t)\Big|_0^{1/8} = \underline{\mathbf{-\,5.4\ J}}$$

Problem 6.4 Find $v_c(t)$, as shown in Figure 6.3, given that



(a)



(b)

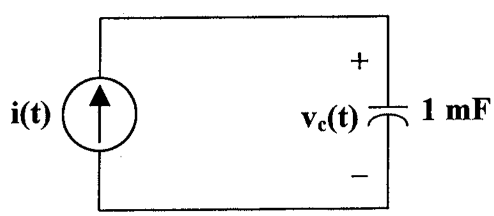


Figure 6.3

(a) $$v_c(t) = \begin{cases} \mathbf{0\ V} & \mathbf{t<0} \\ \mathbf{\frac{t^2}{2}\ kV} & \mathbf{0<t<2} \\ \mathbf{(2t-2)\ kV} & \mathbf{2<t<5} \\ \mathbf{(-t^2+12t-27)\ kV} & \mathbf{5<t<6} \\ \mathbf{9\ kV} & \mathbf{6<t} \end{cases}$$

(b) $$v_c(t) = \begin{cases} \mathbf{0\ V} & \mathbf{t<-2} \\ \mathbf{(4t+8)\ kV} & \mathbf{-2<t<0} \\ \mathbf{(-4t+8)\ kV} & \mathbf{0<t<2} \\ \mathbf{0\ V} & \mathbf{2<t} \end{cases}$$

SERIES AND PARALLEL CAPACITORS

Problem 6.5 Given the circuit in Figure 6.4, $v_1(0^-) = 0$ V, $v_2(0^-) = 100$ V, calculate the voltages after the switch closes.

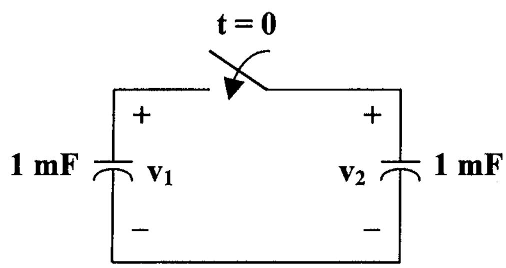


Figure 6.4

- **Carefully DEFINE the problem.**
  Each component is labeled completely. The problem is clear.

- **PRESENT everything you know about the problem.**
  Since the capacitors are in parallel, the charge, $q = CV$, must remain the same.

  Also, when the switch is closed, the voltages across the capacitors are the same.

- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
  The three solution techniques that can be used are nodal analysis, mesh analysis, and basic circuit analysis. Basic circuit analysis can be used to solve this problem.

- **ATTEMPT a problem solution.**

For $t < 0$, $w = \frac{1}{2}C_2V_2^2 = \frac{1}{2}(10^{-3})(100)^2 = 5$ J (there is no initial charge on C_1)

For $t < 0$, $q = C_2V_2 = (10^{-3})(100) = 0.1$ C

For $t > 0$, $C = C_1 + C_2 = 2\,\text{mF}$

$$V = \frac{0.1}{2\times10^{-3}} = 50\text{ V}$$

Now, $w = \frac{1}{2}CV^2 = \frac{1}{2}(2\times10^{-3})(50)^2 = 2.5$ J

Clearly, the energy has gone from 5 J to 2.5 J. What happened to 2.5 J of energy? Well, the switch cannot close fast enough to keep from having a spark. Thus, 2.5 J of energy must be dissipated in the spark.

➢ **EVALUATE the solution and check for accuracy.**
After the switch closes, the charge of 0.1 C remains the same and the voltage across both is now the same, 50 V.
Our check for accuracy was successful.

➢ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

$$v_1 = v_2 = \underline{\mathbf{50\ V}}$$

Problem 6.6 Find the equivalent capacitance for the collection of capacitors shown in Figure 6.5.

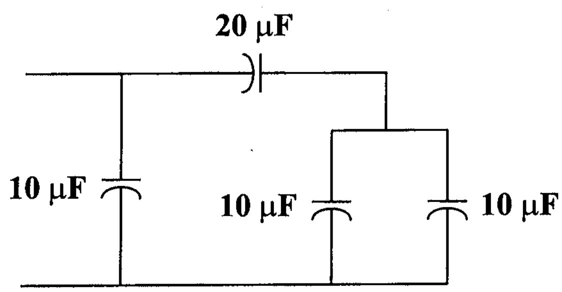


Figure 6.5

$$10 \text{ in parallel with } 10 = 10 + 10 = 20$$
$$20 \text{ in series with } 20 = \frac{(20)(20)}{20+20} = 10$$
$$10 \text{ in parallel with } 10 = 10 + 10 = 20$$

Therefore, $C_{eq} = \underline{\mathbf{20\ \mu F}}$

Problem 6.7 Given that the equivalent capacitance of the collection of capacitors shown in Figure 6.6 is $30\ \mu F$, find C_1 and C_2.

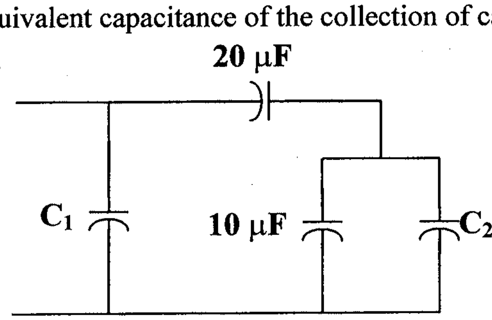


Figure 6.6

There is an infinite number of solutions.
To find one solution, let $C_2 = 10\ \mu F$, the network is similar to the one shown in Problem 6.6.
The first two combinations are the same. Hence, we have

$$10 \text{ in parallel with } C_1 = C_{eq}$$

or $$10 + C_1 = 30 \longrightarrow C_1 = 20$$

Therefore, $C_1 = \underline{\mathbf{20\ \mu F}}$ and $C_2 = \underline{\mathbf{10\ \mu F}}$ produce $C_{eq} = 30\ \mu F$

Problem 6.8 *[6.17]* Calculate the equivalent capacitance for the circuit in Figure 6.7. All capacitances are in mF.

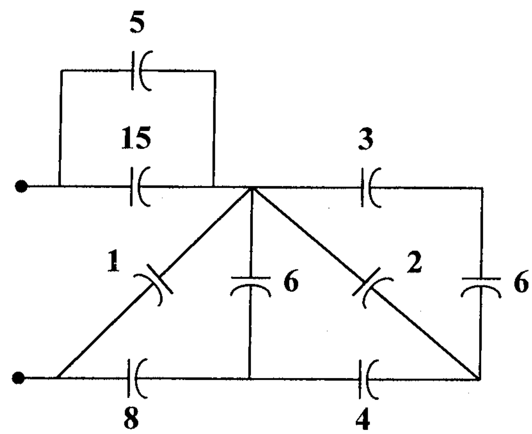
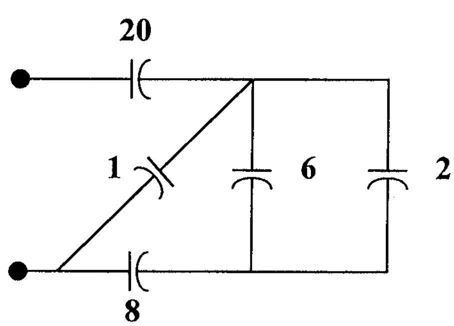


Figure 6.7

$$3 \text{ in series with } 6 = \frac{(6)(3)}{6+3} = 2$$
$$2 \text{ in parallel with } 2 = 2 + 2 = 4$$
$$4 \text{ in series with } 4 = \frac{(4)(4)}{4+4} = 2$$

The circuit is reduced to that shown below:



$$6 \text{ in parallel with } 2 = 6+2 = 8$$

$$8 \text{ in series with } 8 = \frac{(8)(8)}{8+8} = 4$$

$$4 \text{ in parallel with } 1 = 4+1 = 5$$

$$5 \text{ in series with } 20 = \frac{(5)(20)}{5+20} = 4$$

Therefore, $C_{eq} = \underline{\mathbf{4\ mF}}$

INDUCTORS

Problem 6.9 For the circuit shown in Figure 6.8,

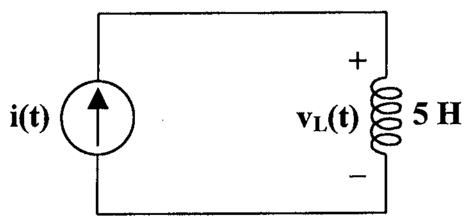


Figure 6.8

calculate $v_L(t)$ given that

(a) $i(t) = (5t+6)$ A

(b) $i(t) = 3\sin(\omega t + 30)$ A

(a) $$v_L(t) = L\frac{di(t)}{dt} = 5\frac{d(5t+6)}{dt} = \underline{\mathbf{25\ V}}$$

(b) $$v_L(t) = L\frac{di(t)}{dt} = 5\frac{d(3\sin(\omega t + 30°))}{dt}$$

$$v_L(t) = \underline{\mathbf{15\omega\cos(\omega t + 30°)\ V}}$$

Problem 6.10 For the circuit shown in Figure 6.9, calculate $v_L(t)$ given that

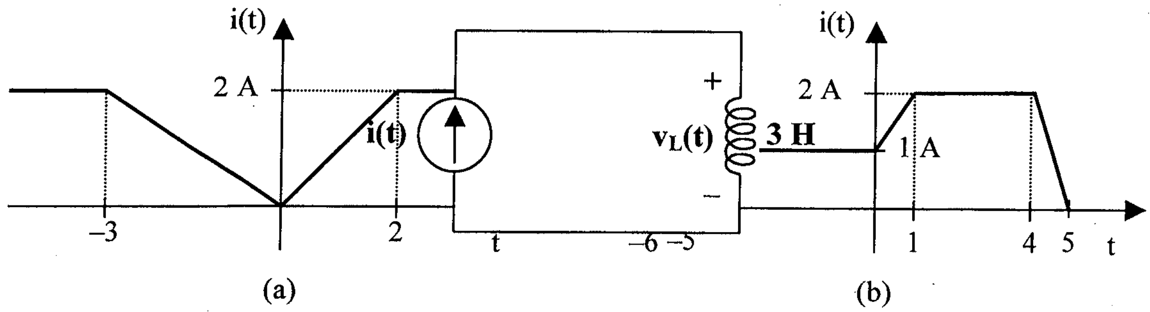


Figure 6.9

$$v_L(t) = L\frac{di_L(t)}{dt}$$

(a) For $t < -3$, $\quad v_L(t) = 3\frac{d(2)}{dt} = \underline{\mathbf{0\ V}}$

For $-3 < t < 0$, $\quad v_L(t) = 3\frac{d(-2t/3)}{dt} = \underline{\mathbf{-2\ V}}$

For $0 < t < 2$, $\quad v_L(t) = 3\frac{d(t)}{dt} = \underline{\mathbf{3\ V}}$

For $2 < t$, $\quad v_L(t) = 3\frac{d(2)}{dt} = \underline{\mathbf{0\ V}}$

(b) For $t < -6$, $\quad v_L(t) = 3\frac{d(0)}{dt} = \underline{\mathbf{0\ V}}$

For $-6 < t < -5$, $\quad v_L(t) = 3\frac{d(t+6)}{dt} = \underline{\mathbf{3\ V}}$

For $-5 < t < 0$, $\quad v_L(t) = 3\frac{d(1)}{dt} = \underline{\mathbf{0\ V}}$

For $0 < t < 1$, $\quad v_L(t) = 3\frac{d(t+1)}{dt} = \underline{\mathbf{3\ V}}$

For $1 < t < 4$, $\quad v_L(t) = 3\frac{d(2)}{dt} = \underline{\mathbf{0\ V}}$

For $4 < t < 5$, $\quad v_L(t) = 3\dfrac{d(-2t+10)}{dt} = \underline{\mathbf{-6\ V}}$

For $5 < t$, $\quad v_L(t) = 3\dfrac{d(0)}{dt} = \underline{\mathbf{0\ V}}$

Problem 6.11 *[6.35]* The voltage across a 2-H inductor is $20(1 - e^{-2t})$ V. If the initial current through the inductor is 0.3 A, find the current and the energy stored in the inductor at $t = 1$ s.

$$i = \frac{1}{L}\int_0^t v\,dt + i(0) = \frac{1}{2}\int_0^t (20)(1 - e^{-2t})\,dt + 0.3$$

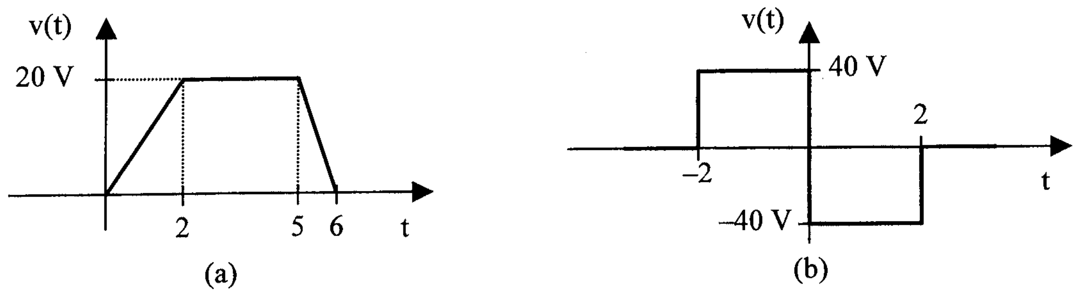
$$i = (10)\left(t + \frac{1}{2}e^{-2t}\right)\Big|_0^t + 0.3 = (10t + 5e^{-2t} - 4.7)\text{ A}$$

At $t = 1$ s,

$$i = 10 + 5e^{-2} - 4.7 = \underline{\mathbf{5.977\ A}}$$

$$w = \frac{1}{2}Li^2 = \underline{\mathbf{35.72\ J}}$$

Problem 6.12 For the circuit shown in Figure 6.10, calculate $i_L(t)$ given that



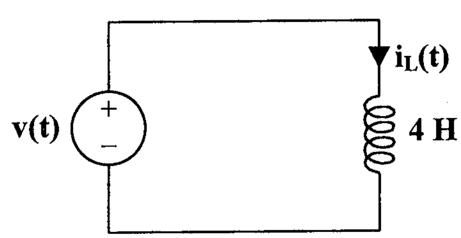


Figure 6.10

(a) $i_L(t) = \begin{cases} 0 \text{ A} & t < 0 \\ (5/4)t^2 \text{ A} & 0 < t < 2 \\ (5t - 5) \text{ A} & 2 < t < 5 \\ (-2.5t^2 + 30t - 67.5) \text{ A} & 5 < t < 6 \\ 22.5 \text{ A} & 6 < t \end{cases}$ (b) $i_L(t) = \begin{cases} 0 \text{ A} & t < -2 \\ (10t + 20) \text{ A} & -2 < t < 0 \\ (-10t + 20) \text{ A} & 0 < t < 2 \\ 0 \text{ A} & 2 < t \end{cases}$

SERIES AND PARALLEL INDUCTORS

Problem 6.13 Given the collection of inductors shown in Figure 6.11, find the value of the equivalent inductance.

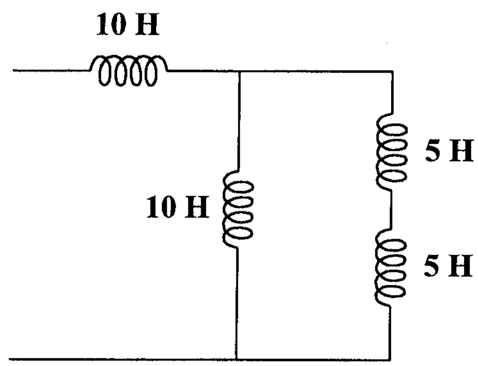


Figure 6.11

$$5 \text{ in series with } 5 = 5 + 5 = 10$$
$$10 \text{ in parallel with } 10 = \frac{(10)(10)}{10 + 10} = 5$$
$$5 \text{ in series with } 10 = 5 + 10 = 15$$

Thus, $L_{eq} = \underline{\mathbf{15\ H}}$

Problem 6.14 Given the collection of inductors shown in Figure 6.12, find the values of L_1 and L_2, when the equivalent inductance is 20 H.

Problem 6.15

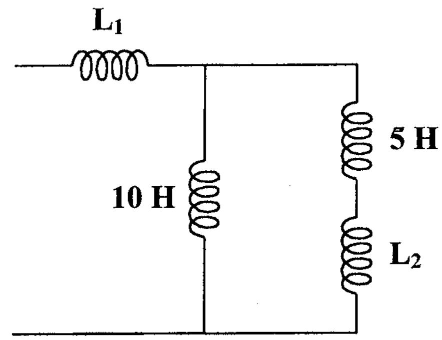


Figure 6.12

There is an infinite number of solutions.

However, if $L_2 = 5\ \text{H}$, the network is similar to the one shown in Problem 6.13. The first two combinations are the same. Hence, we have

$$5 \text{ in series with } L_1 = L_{eq}$$

or $$5 + L_1 = 20 \longrightarrow L_1 = 15$$

Therefore, $L_1 = \underline{\mathbf{15\ H}}$ and $L_2 = \underline{\mathbf{5\ H}}$ produce $L_{eq} = 20\ \text{H}$

APPLICATIONS

Problem 6.16 Calculate the voltage across the current source in Figure 6.13 given that $i(t) = (t + 5)\ \text{A}$.

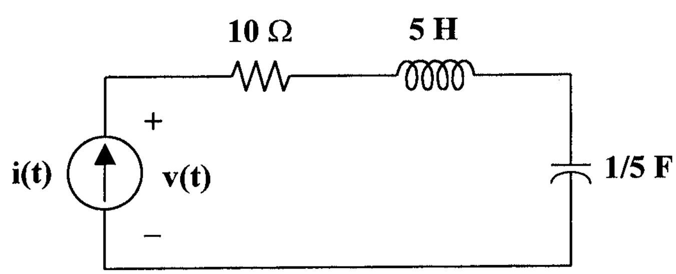


Figure 6.13

- **Carefully DEFINE the problem.**
  Each component is labeled completely. The problem is clear, except for the value of the capacitor voltage at some point in time.

- **PRESENT everything you know about the problem.**
We know the current as well as the values of the elements. However, we do not know the initial condition on the voltage across the capacitor. We will solve for the voltage across the current source assuming that the capacitor voltage at $t = 0$ is equal to $v_C(0)$.

- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
The three solution techniques that can be used are nodal analysis, mesh analysis, and basic circuit analysis. Basic circuit analysis will be used to solve this problem.

- **ATTEMPT a problem solution.**

$$v(t) = v_R(t) + v_L(t) + v_C(t)$$

$$v(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(\tau)\, d\tau$$

$$v(t) = (10)(t+5) + (5)(1) + 5\int_0^t (\tau+5)\, d\tau + v_C(0)$$

$$v(t) = 10t + 50 + 5 + (5)\left(\frac{t^2}{2} + 5t\right) + v_C(0)$$

$$v(t) = 10t + 50 + 5 + 2.5t^2 + 25t + v_C(0)$$

$$v(t) = [2.5t^2 + 35t + 55]\text{ V} + v_C(0)$$

- **EVALUATE the solution and check for accuracy.**
The current through each element is the same. The voltage across each element was determined while attempting a problem solution.

For the resistor, $i_R(t) = \dfrac{v_R(t)}{10} = \dfrac{10t+50}{10} = t+5$

For the inductor, $i_L(t) = \dfrac{1}{L}\displaystyle\int v_L(\tau)\, d\tau = \dfrac{1}{5}\int_0^t 5\, d\tau = \dfrac{1}{5}\cdot 5t + i_L(0) = t + i_L(0)$

For the capacitor, $i_C(t) = C\dfrac{dv_C(t)}{dt} = \dfrac{1}{5}\cdot\dfrac{d}{dt}(2.5t^2 + 25t) = \dfrac{1}{5}(5t+25) = t+5$

Hence,

$$i(t) = i_R(t) = i_L(t) = i_C(t), \qquad \text{when} \quad i(0) = 5 = i_L(0)$$

Our check for accuracy was successful.

- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

$$v(t) = \underline{\mathbf{[2.5t^2 + 35t + 55]\ V + v_C(0)}}$$

Problem 6.17 Given the circuit in Figure 6.14, find $v(t)$ for $v_C(t) = (10 + 5t)$ V.

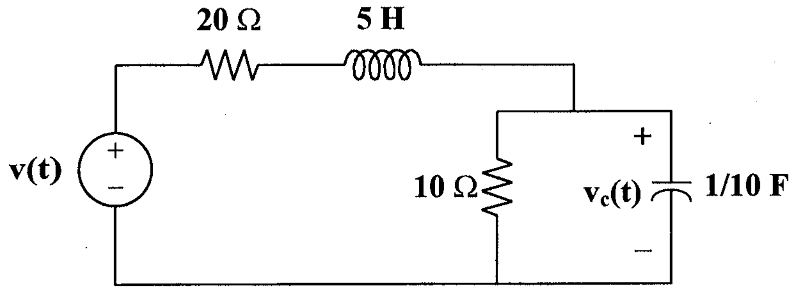


Figure 6.14

$$i_c(t) = C\frac{dv_C(t)}{dt} = \frac{1}{10}\frac{d}{dt}(10+5t) = \left(\frac{1}{10}\right)(5) = \frac{1}{2}\text{ A}$$

$$i_{10\Omega}(t) = \frac{v_C(t)}{R} = \frac{10+5t}{10} = \left(1 + \frac{1}{2}t\right)\text{ A}$$

$$i_L(t) = \frac{1}{2} + \left(1 + \frac{1}{2}t\right) = \left(\frac{3}{2} + \frac{1}{2}t\right)\text{ A}$$

$$v_L(t) = L\frac{di(t)}{dt} = (5)\frac{d}{dt}\left(\frac{3}{2} + \frac{1}{2}t\right) = (5)\left(\frac{1}{2}\right) = 2.5\text{ V}$$

$$v_{20\Omega}(t) = i_{20\Omega}(t)R = i_L(t)R = \left(\frac{3}{2} + \frac{1}{2}t\right)(20) = (30+10t)\text{ V}$$

$$v(t) = v_{20\Omega}(t) + v_L(t) + v_C(t) = (30+10t) + (2.5) + (10+5t)$$
$$v(t) = \underline{\mathbf{(15t + 42.5)\ V}}$$

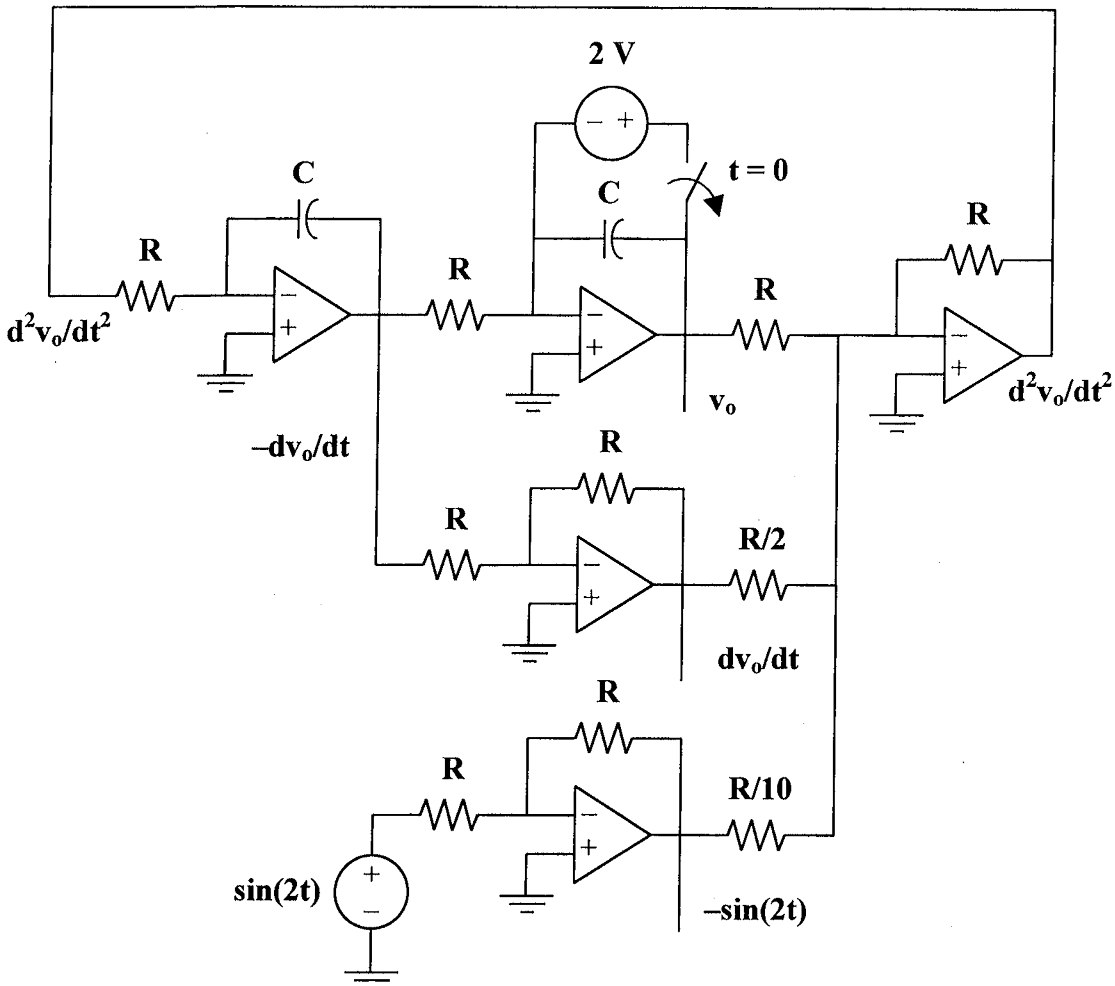
Problem 6.18 *[6.67]* Design an analog computer to simulate

$$\frac{d^2v_o}{dt} + 2\frac{dv_o}{dt} + v_o = 10\sin(2t)$$

where $v_o(0) = 2$ and $v_o'(0) = 0$.

$$\frac{d^2v_o}{dt} = 10\sin(2t) - 2\frac{dv_o}{dt} - v_o$$

Thus, by combining integrators with a summer, we obtain the appropriate analog computer as shown below.



Problem 6.19 Calculate $v(t)$ and $v_L(t)$ for the circuit shown in Figure 6.15 and $v_C(0) = -10$ Volts (with the plus side of v_C at the top of the capacitor).

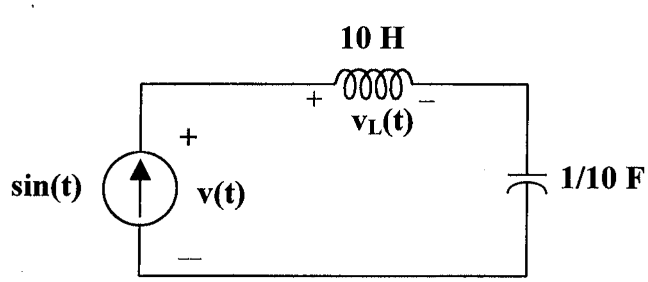


Figure 6.15

$v(t) =$ **0 V** $\qquad\qquad$ $v_L(t) =$ **10cos(t) V**

CHAPTER 7 - FIRST-ORDER CIRCUITS

List of topics for this chapter :

SOURCE-FREE RC CIRCUIT

Problem 7.1 For the circuit in Figure 7.1, find $v_C(t)$ and $i_C(t)$ given $v_C(0) = 10$ V .

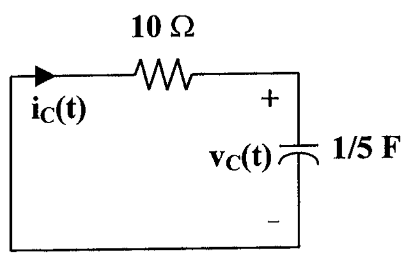


Figure 7.1

➢ **Carefully DEFINE the problem.**
Each component is labeled completely. The problem is clear.

➢ **PRESENT everything you know about the problem.**
This is a source-free RC circuit. The natural response of this source-free RC circuit is

$$v_C(t) = V_0\, e^{-t/\tau}, \qquad \text{where} \quad V_0 = v_C(0) \quad \text{and} \quad \tau = RC$$

We know the initial voltage across the capacitor. To find the capacitor voltage for any time greater than zero, we need to calculate the time constant of the circuit.

➢ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
The three solution techniques that can be used are nodal analysis, mesh analysis, and basic circuit analysis. Basic circuit analysis can be used to solve this problem.

➢ **ATTEMPT a problem solution.**

$V_0 = v_C(0) = 10\text{ V}$ and $\tau = RC = (10)(1/5) = 2\text{ s}$

$v_C(t) = 10e^{-t/2}\text{ V}$

$$i_C(t) = \frac{-v_C(t)}{R} \quad \text{or} \quad i_C(t) = C\frac{dv_C(t)}{dt}$$

In either case, $i_C(t) = -e^{-t/2}\text{ A}$

➢ **EVALUATE the solution and check for accuracy.**
Using KVL,

$$10i_C(t) + v_C(t) = (10)(-e^{-t/2}) + 10e^{-t/2} = 0$$

Our check for accuracy was successful.

➢ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

$i_C(t) = \underline{\boldsymbol{-e^{-t/2}}\textbf{ amps}}$ for all $t > 0$.

Problem 7.2 *[7.23]* Express the signals in Figure 7.2 in terms of singularity functions.

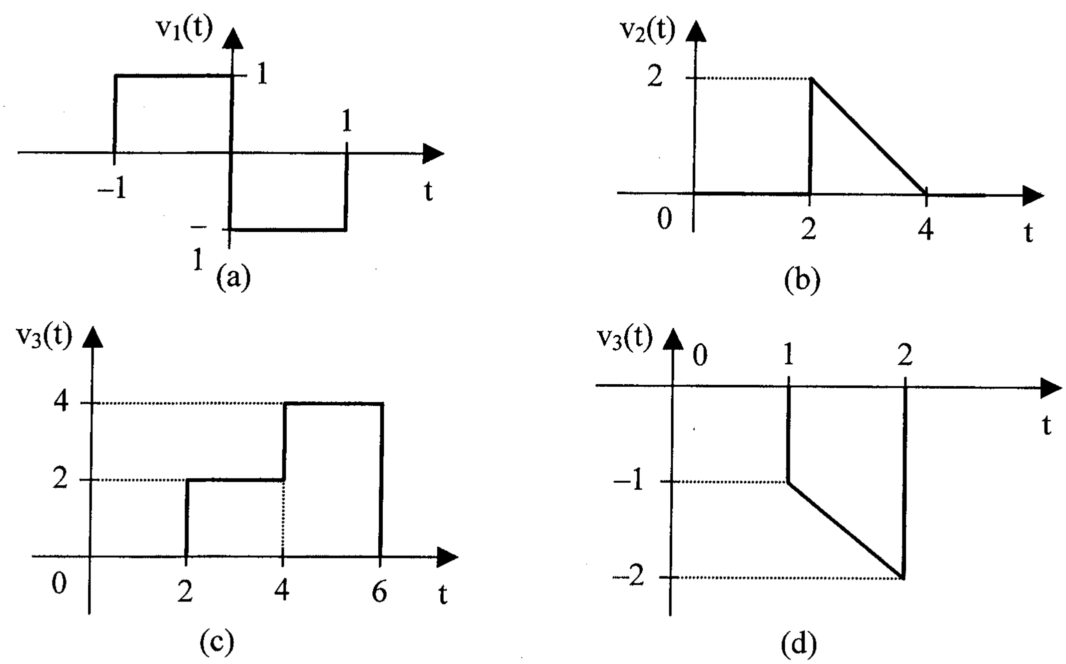


Figure 7.2

(a) $$v_1(t) = u(t+1) - u(t) + [u(t-1) - u(t)]$$
$$v_1(t) = \underline{\mathbf{u(t+1) - 2u(t) + u(t-1)}}$$

(b) $$v_2(t) = (4-t)[u(t-2) - u(t-4)]$$
$$v_2(t) = -(t-4)u(t-2) + (t-4)u(t-4)$$
$$v_2(t) = \underline{\mathbf{2u(t-2) - r(t-2) + r(t-4)}}$$

(c) $$v_3(t) = 2[u(t-2) - u(t-4)] + 4[u(t-4) - u(t-6)]$$
$$v_3(t) = \underline{\mathbf{2u(t-2) + 2u(t-4) - 4u(t-6)}}$$

(d) $$v_4(t) = -t[u(t-1) - u(t-2)] = -t\,u(t-1) + t\,u(t-2)$$
$$v_4(t) = (-t+1-1)u(t-1) + (t-2+2)u(t-2)$$
$$v_4(t) = \underline{\mathbf{-r(t-1) - u(t-1) + r(t-2) + 2u(t-2)}}$$

Problem 7.3 Given $i(t) = 3e^{-t/2}$ A, find $v_C(t)$ for the circuit shown in Figure 7.3.

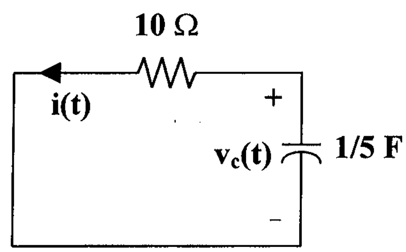


Figure 7.3

$$v_C(t) = \underline{\mathbf{30e^{-t/2}\ V}}$$

Problem 7.4 Given $v_C(1) = 10$ V, find $v_C(t)$ for all $t > 0$ in Figure 7.4.

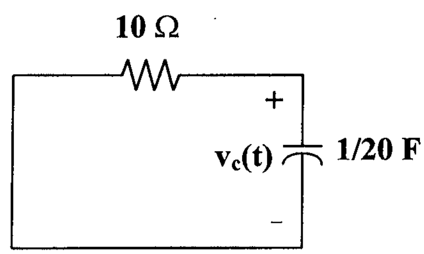


Figure 7.4

$$v_C(t) = \underline{\mathbf{\left(\frac{10}{e^{-2}}\right)e^{-2t}\ V}}$$

SOURCE-FREE RL CIRCUIT

Problem 7.5 For the circuit in Figure 7.5, find $i(t)$ and $v_L(t)$ given $i(0) = 4\text{ A}$.

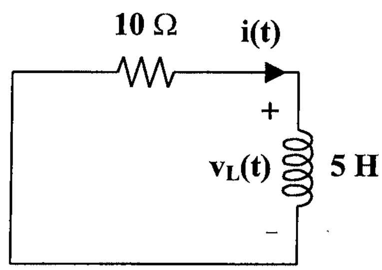


Figure 7.5

- **Carefully DEFINE the problem.**
  Each component is labeled completely. The problem is clear.

- **PRESENT everything you know about the problem.**
  This is a source-free RL circuit. The natural response of this source-free RL circuit is

$$i_L(t) = I_0\, e^{-t/\tau}, \qquad \text{where} \quad I_0 = i_L(0) \qquad \text{and} \qquad \tau = L/R$$

We know the initial current through the inductor. To find the current through the inductor for any value of time greater than zero, we need to calculate the time constant of the circuit.

- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
  The three solution techniques that can be used are nodal analysis, mesh analysis, and basic circuit analysis. Basic circuit analysis can be used to solve this problem.

- **ATTEMPT a problem solution.**

$$I_0 = i_L(0) = 4\text{ A} \qquad \text{and} \qquad \tau = L/R = 5/10 = 0.5\text{ s}$$

$$i(t) = 4e^{-2t}\text{ A}$$

$$v_L(t) = -10i(t) \qquad \text{or} \qquad v_L(t) = L\frac{di(t)}{dt}$$

In either case,

$$v_L(t) = -40e^{-2t}\text{ V}$$

➢ **EVALUATE the solution and check for accuracy.**
Using KVL,

$$10i(t) + v_L(t) = (10)(4e^{-2t}) - 40e^{-2t} = 0$$

Our check for accuracy was successful.

➢ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

$$i(t) = \underline{\mathbf{4e^{-2t}u(t)\ A}} \qquad \text{and} \qquad v_L(t) = \underline{\mathbf{-40e^{-2t}u(t)\ V}}$$

Problem 7.6 For the circuit in Figure 7.6, find $i(t)$ given $v_L(t) = 20e^{-2t}$ V.

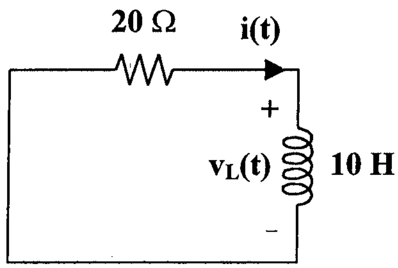


Figure 7.6

$$L/R = 10/20 = 0.5 \text{ s}$$

$$i(t) = \frac{1}{L}\int v_L(\tau)\, d\tau$$

but it is also

$$i(t) = \frac{-v_L(t)}{R} = \underline{\mathbf{-e^{-2t}u(t)\ A}}$$

Problem 7.7 Given $i(0) = 2$ A, find $i(t)$, $p_{10\Omega}$ (power absorbed by the 10 ohm resistor), and $w_{10\Omega}$ (total energy dissipated by the 10 ohm resistor) for the circuit in Figure 7.7.

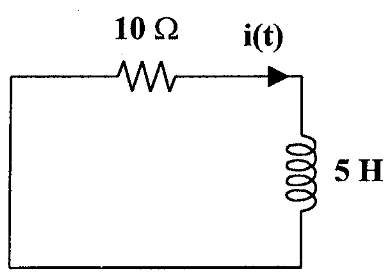


Figure 7.7

$$i(t) = \underline{\mathbf{2\,e^{-2t}\ A}}$$

$$p_{10\Omega} = \underline{\mathbf{40\,e^{-4t}\ W}} \qquad\qquad w_{10\Omega} = \underline{\mathbf{10\ J}}$$

SINGULARITY FUNCTIONS

Problem 7.8 Solve for

(a) $\dfrac{du(t)}{dt}$ (b) $\dfrac{dr(t)}{dt}$

(a) $u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$

$$\frac{d}{dt}u(t) = \begin{cases} 0 & t < 0 \\ \text{undefined} & t = 0 \\ 0 & t > 0 \end{cases} \longrightarrow \frac{d}{dt}u(t) = \underline{\boldsymbol{\delta(t)}}$$

(b) $r(t) = \begin{cases} 0 & t \le 0 \\ t & t > 0 \end{cases}$

$$\frac{d}{dt}r(t) = \begin{cases} 0 & t \le 0 \\ 1 & t > 0 \end{cases} \longrightarrow \frac{d}{dt}r(t) = \underline{\mathbf{u(t)}}$$

Problem 7.9 Given $v_C(t) = [5\,u(t) + 6\,r(t)]$ V, find $i_C(t)$ for the circuit in Figure 7.8.

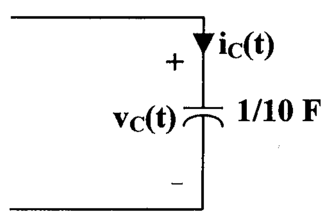


Figure 7.8

$$i_C(t) = C\frac{d}{dt}v_C(t)$$

$$i_C(t) = \frac{1}{10} \cdot \frac{d}{dt}[5u(t) + 6r(t)] = \frac{1}{10} \cdot \left[5\frac{d}{dt}u(t) + 6\frac{d}{dt}r(t)\right]$$

Using Problem 7.8, it is clear that

$$i_C(t) = \underline{\mathbf{\frac{1}{10}[5\delta(t) + 6u(t)]\ A}}$$

Problem 7.10 Solve for

(a) $\int \delta(t)\, dt$ (b) $\int u(t)\, dt$

(a) $\underline{\mathbf{u(t)}}$ (b) $\underline{\mathbf{r(t)}}$

STEP RESPONSE OF AN RC CIRCUIT

Problem 7.11 Given $v(t) = 20u(t)$ V, find $v_C(t)$ and $i_C(t)$ in Figure 7.9.

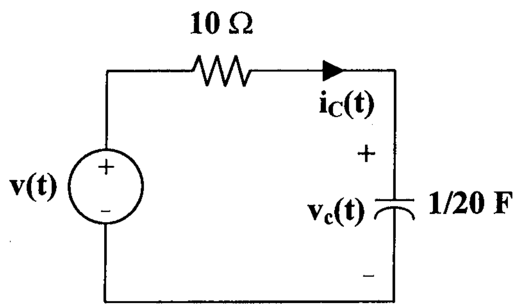


Figure 7.9

$$\tau = RC = (10)\left(\frac{1}{20}\right) = \frac{1}{2}\ s$$

$$v_C(0) = 0\ V \qquad v_C(\infty) = 20\ V$$

$$v_C(t) = \underline{\mathbf{(20)(1 - e^{-2t})u(t)\ V}}$$

$$i_C(t) = C\frac{dv_C(t)}{dt} = \left(\frac{1}{20}\right)(-20)(-2e^{-2t}) = \underline{\mathbf{2e^{-2t}\ u(t)\ A}}$$

Problem 7.12 *[7.37]* Find the step responses $v(t)$ and $i(t)$ to $v_s = 5u(t)$ V in the circuit of Figure 7.10

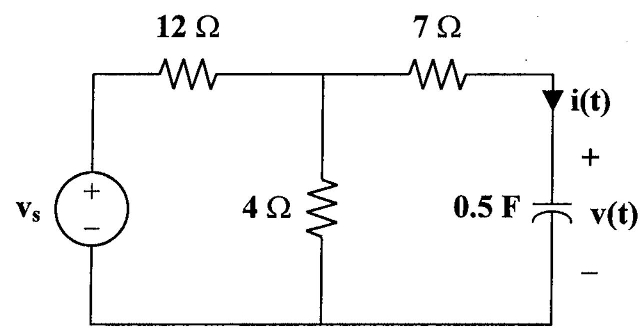


Figure 7.10

For $t < 0$, $v_s = 5u(t) = 0 \longrightarrow v(0) = 0$ V

For $t > 0$, $v_s = 5$ V, $v(\infty) = \dfrac{4}{4+12}(5) = \dfrac{5}{4}$ V

$R_{eq} = 7 + 4 \,||\, 12 = 10\,\Omega$, $\tau = R_{eq}C = (10)(1/2) = 5$ s

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$
$$v(t) = \underline{\mathbf{1.25(1 - e^{-t/5})\ V}}$$

$$i(t) = C\frac{dv}{dt} = \left(\frac{1}{2}\right)\left(\frac{-5}{4}\right)\left(\frac{-1}{5}\right) e^{-t/5}$$
$$i(t) = \underline{\mathbf{0.125 e^{-t/5}\ A}}$$

Problem 7.13 Given $v(t) = 10[u(t) - u(t-2)]$ V, find $v_C(t)$ in Figure 7.11.

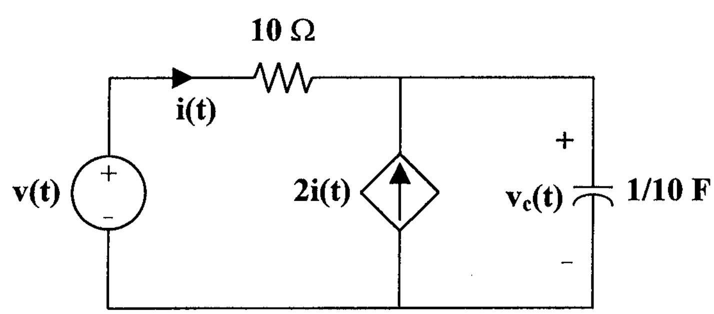
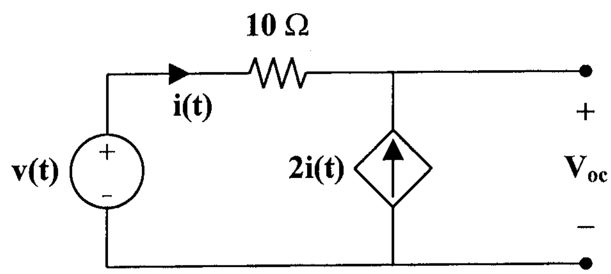


Figure 7.11

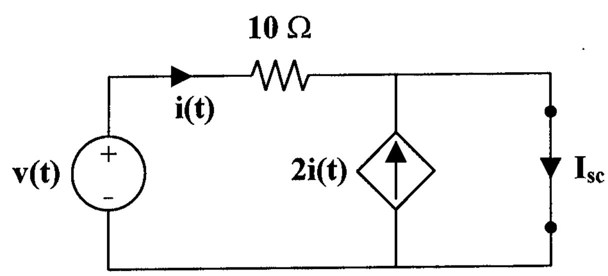
Find the Thevenin equivalent of the circuit at the terminals of the capacitor. This will simplify the circuit, forming an RC circuit with a voltage source.

Use the following circuit to find the open circuit voltage.



V_{oc} must be equal to $v(t)$, since $i(t) + 2i(t) = 0 \longrightarrow i(t) = 0\text{ A}$.

To find the short circuit current,



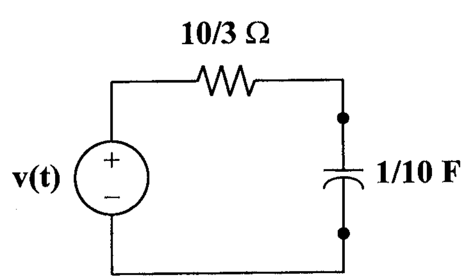
$$I_{sc} = i(t) + 2i(t) = 3i(t) \qquad \text{where } i(t) = \frac{v(t)}{10}$$

$$I_{sc} = \frac{3}{10}v(t)$$

Thus,

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{10}{3}\,\Omega$$

which leads to the following Thevenin equivalent circuit.



Using the Thevenin equivalent circuit with the capacitor as the load, we can see that

$$\tau = R_{th}C = \left(\frac{10}{3}\right)\left(\frac{1}{10}\right) = \frac{1}{3}\text{ s}$$

For $t < 0$, $v(t) = 0\text{ V}$, $v_C(0) = 0\text{ V}$, $v_C(\infty) = 0\text{ V}$

$$v_C(t) = 0\text{ V}$$

For $0 < t < 2$, $v(t) = 10\text{ V}$, $v_C(0) = 0\text{ V}$, $v(\infty) = 10\text{ V}$

$$v_C(t) = (10)(1 - e^{-3t})\text{ V}$$

For $2 < t$, $v(t) = 0\text{ V}$, $v_C(2) = (10)(1 - e^{-6})\text{ V}$, $v_C(\infty) = 0\text{ V}$

$$v_C(t) = (10)(1 - e^{-6})e^{-3(t-2)}\text{ V}$$

Combining these cases,

$$v_C(t) = \underline{\left\{ \left[\mathbf{(10)(1 - e^{-3t})[u(t) - u(t-2)]} \right] + \left[\mathbf{9.975\, e^{-3(t-2)}\, u(t-2)} \right] \right\} \mathbf{V}}$$

STEP RESPONSE OF AN RL CIRCUIT

Problem 7.14 Given $v(t) = 40\,u(t)\text{ V}$, find $i_L(t)$ and $v_L(t)$ in Figure 7.12.

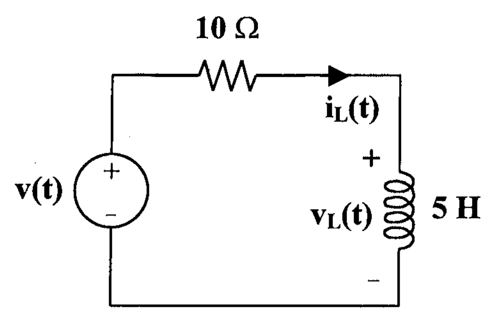


Figure 7.12

$$\tau = \frac{L}{R} = \frac{5}{10} = \frac{1}{2}\text{ s}$$

$$i_L(0) = 0\text{ A}$$

$$i_L(t) = \underline{\mathbf{(4)(1 - e^{-2t})\, u(t)\ A}}$$

$$v_L(t) = L\frac{di_L(t)}{dt} = (5)(4)(2)e^{-2t}\,u(t) = \underline{\mathbf{40\,e^{-2t}\,u(t)\ V}}$$

Problem 7.15 *[7.55]* Find $v_o(t)$ for $t > 0$ in the circuit of Figure 7.13.

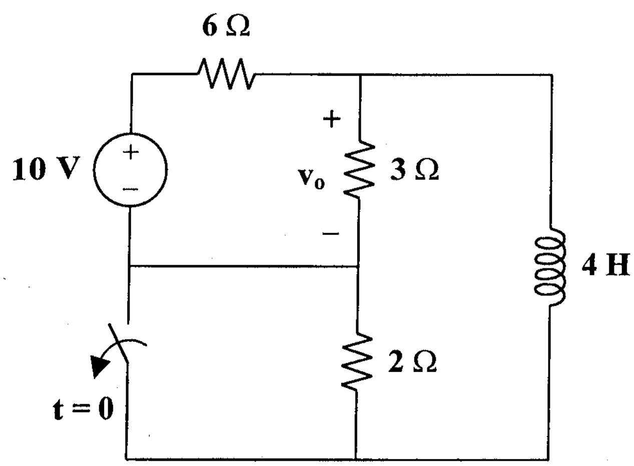
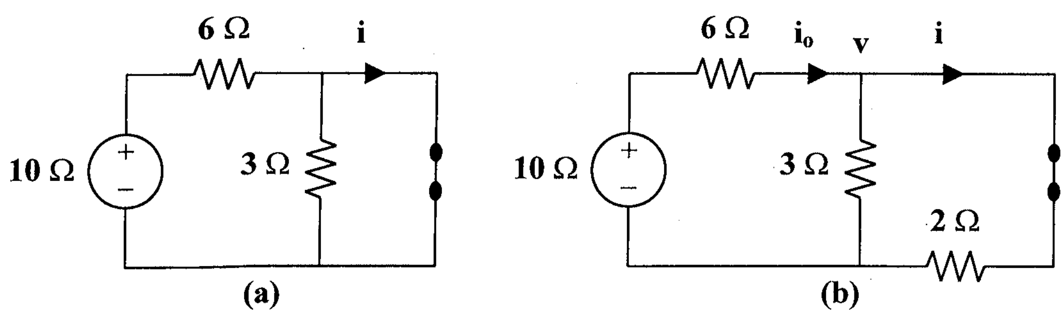


Figure 7.13

Let i be the inductor current. For t < 0, the inductor acts like a short circuit and the 2 Ω resistor is short-circuited so that the equivalent circuit is shown in Fig. (a).



$$i = i(0) = \frac{10}{6} = 1.6667 \text{ A}$$

For $t > 0$, $\quad R_{th} = 2 + 3 \,\|\, 6 = 4\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{4}{4} = 1 \text{ s}$

To find $i(\infty)$, consider the circuit in Fig. (b).

$$\frac{10 - v}{6} = \frac{v}{3} + \frac{v}{2} \longrightarrow v = \frac{10}{6} \text{ V}$$

$$i = i(\infty) = \frac{v}{2} = \frac{5}{6} \text{ A}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = \frac{5}{6} + \left(\frac{10}{6} - \frac{5}{6} \right) e^{-t} = \frac{5}{6}(1 + e^{-t}) \text{ A}$$

v_o is the voltage across the 4 H inductor and the 2 Ω resistor

$$v_o(t) = 2i + L\frac{di}{dt} = \frac{10}{6} + \frac{10}{6}e^{-t} + (4)\left(\frac{5}{6}\right)(-1)e^{-t} = \frac{10}{6} - \frac{10}{6}e^{-t}$$

$$v_o(t) = \underline{\mathbf{1.6667(1 - e^{-t})\ V}}$$

Problem 7.16 Find $i_L(t)$ and $v_L(t)$ in Figure 7.14 for $v(t) = [20u(t) - 40u(t-1)]$ V.

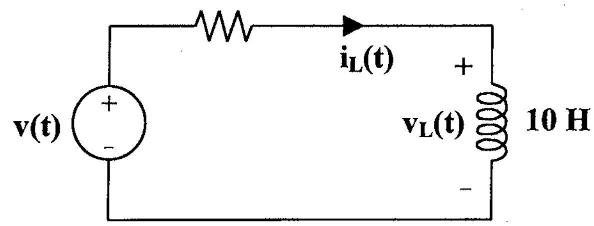


Figure 7.14

$$i_L(t) = \underline{\mathbf{(2)(1 - e^{-t})[u(t) - u(t-1)] + [-2 + (4 - 2e^{-1})e^{-(t-1)}]u(t-1)\ A}}$$

$$v_L(t) = \underline{\mathbf{20e^{-t}u(t) - [20e^{-t} + (20)(2 - e^{-1})e^{-(t-1)}]u(t-1)\ V}}$$

FIRST-ORDER OP AMP CIRCUITS

Problem 7.17 Given $v(t) = 10u(t)$ V, find $i_o(t)$ for the circuit in Figure 7.15.

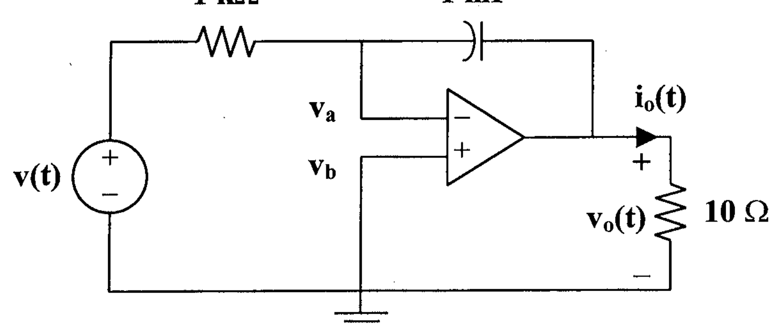


Figure 7.15

$$\frac{v_a - v(t)}{1k} + 1m\frac{d}{dt}[v_a - v_o(t)] = 0, \qquad \text{where } v_a = v_b = 0\text{ V}.$$

$$-v(t) = \frac{dv_o(t)}{dt}$$

$$v_o(t) = -\int v(\tau)\, d\tau = -\int_0^t 10\, u(\tau)\, d\tau = -10t \text{ V}$$

$$i_o(t) = \frac{v_o(t)}{10} = \underline{\mathbf{-t\, u(t)\ A}}$$

Problem 7.18 *[7.59]* Obtain v_o for $t > 0$ in the circuit of Figure 7.16.

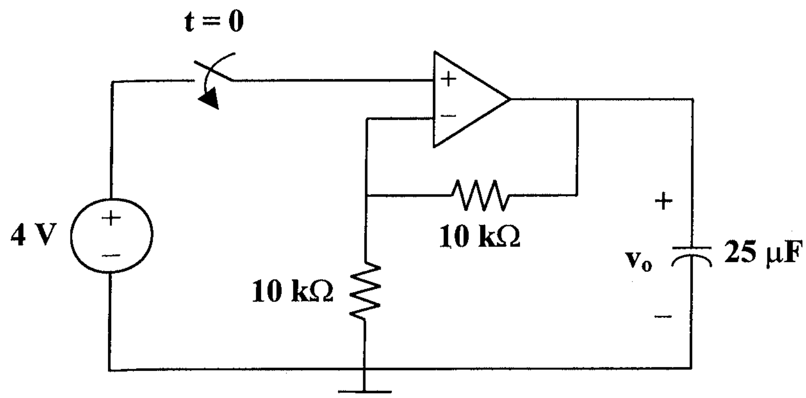


Figure 7.16

This is a very interesting problem and has both an important ideal solution as well as an important practical solution. Let us look at the ideal solution first. Just before the switch closes, the value of the voltage across the capacitor is zero, which means that the voltage at both input terminals of the op amp are zero. As soon as the switch closes, the output tries to go to a voltage such that the inputs to the op amp both go to 4 volts. The ideal op amp puts out whatever current is necessary to reach this condition. An infinite (impulse) current is necessary if the voltage across the capacitor is to go to 8 volts in zero time (8 volts across the capacitor will result in 4 volts appearing at the negative terminal of the op amp). So v_o will be equal to **8 volts** for all $t > 0$.

What happens in a real circuit? Essentially, the output of the amplifier portion of the op amp goes to whatever its maximum value can be. Then, this maximum voltage appears across the output resistance of the op amp and the capacitor that is in series with it. This then results in an exponential rise in the capacitor voltage to the steady-state value of 8 volts.

For all values of $v_C(t)$ less than 8 V,

$$v_C(t) = \underline{\mathbf{V_{op\text{-}amp\text{-}max}\,(1 - e^{-t/(R_{out}C)})\ V}}$$

where $V_{op\text{-}amp\text{-}max}$ is the maximum value of the op amp and R_{out} is the real output resistance of the practical op amp.

When t is large enough so that the 8 V is reached,

$$v_C(t) = \underline{\mathbf{8\ V}}$$

Problem 7.19 *[7.69]* The switch in Figure 7.17 moves from position a to b at $t = 0$. Use PSpice to find $i(t)$ for $t > 0$.

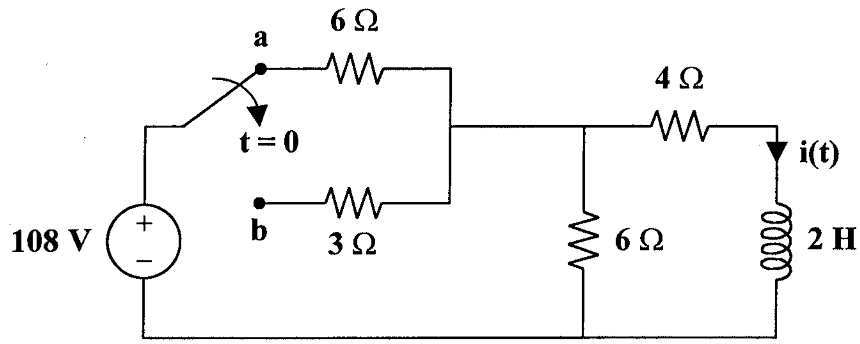
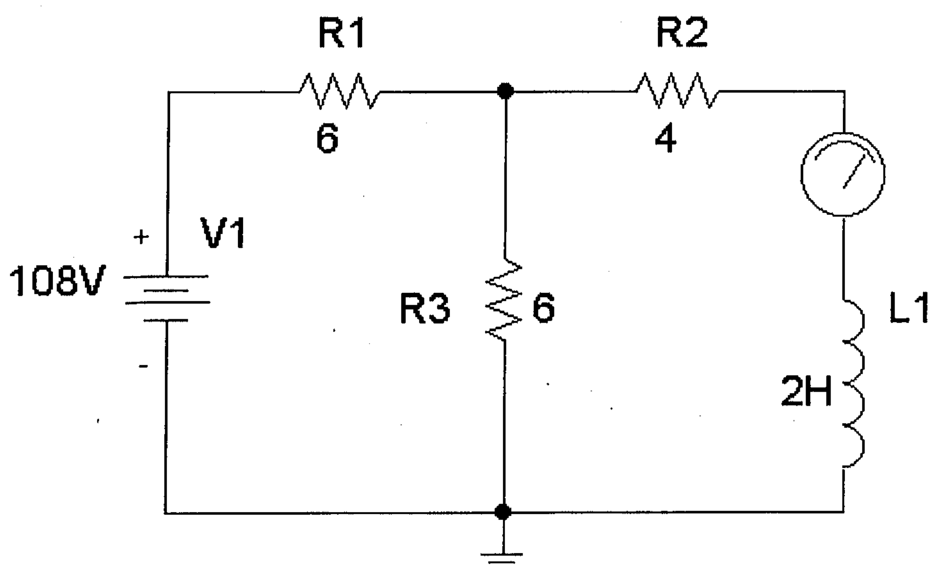


Figure 7.17

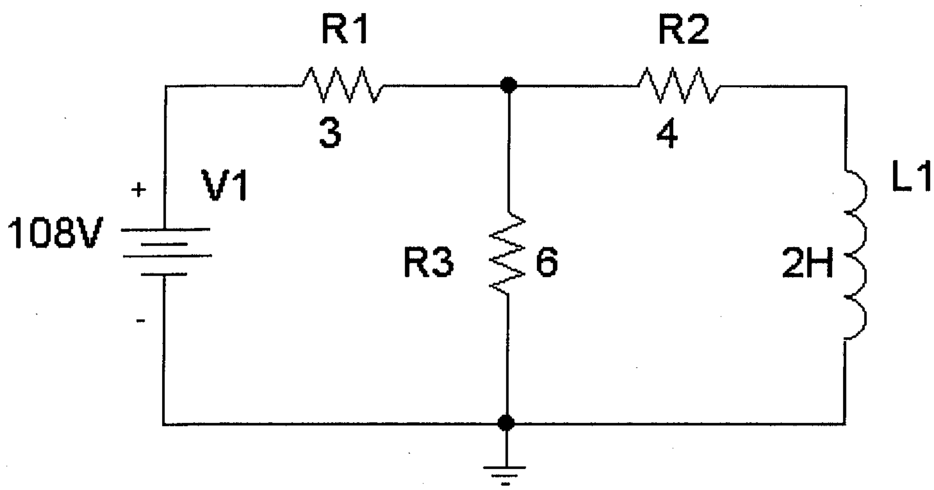
(a) When the switch is in position a, the schematic is shown below. We insert IPROBE to display i. After simulation, we obtain,

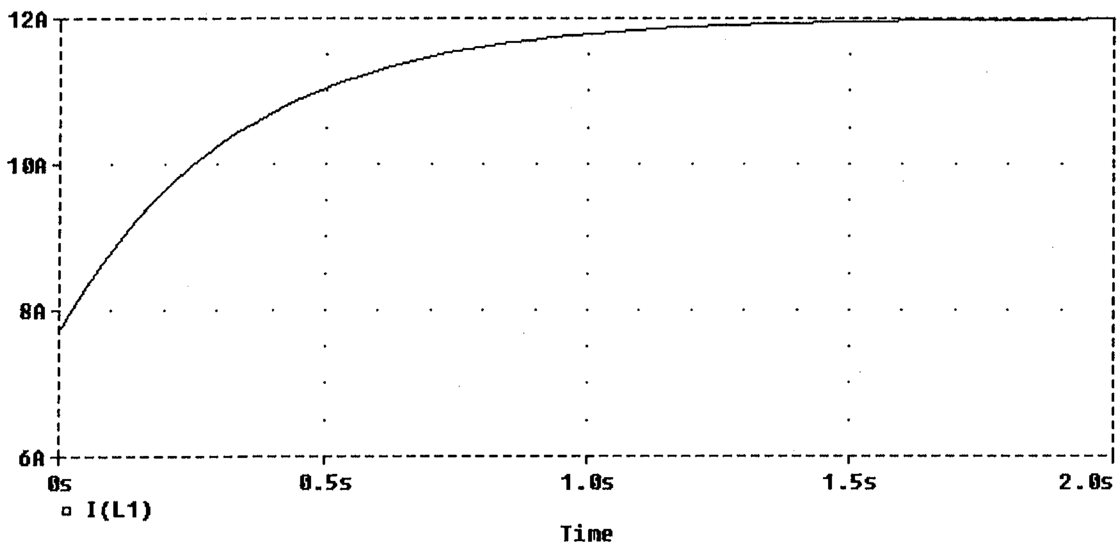
$$i(0) = 7.714 \text{ A}$$

from the display of IPROBE.



(b) When the switch is in position b, the schematic is as shown below. For inductor L1, we let IC = 7.714 A. By clicking Analysis/Setup/Transient, we let Print Step = 25 ms and Final Step = 2 s. After Simulation, we click Trace/Add in the probe menu and display I(L1) as shown below. Note that $i(\infty) = 12$ A, which is correct.





We now know the initial and final values of the current through the inductor.

$$i(0) = 7.714 \text{ A} \qquad\qquad i(\infty) = 12 \text{ A}$$

To find the current through the inductor for any value of time, we need to know the time constant of the circuit. Using the circuit from part (b),

$$R_{eq} = 3 \parallel 6 + 4 = 2 + 4 = 6\ \Omega$$

$$\tau = L/R_{eq} = 2/6 = 1/3 \text{ s}$$

Therefore, $i(t) = i(\infty) + \left[\, i(0) - i(\infty)\right] e^{-t/\tau}$

$$i(t) = 12 + [7.714 - 12]e^{-3t} = \underline{\mathbf{12 - 4.286\,e^{-3t}\ A}}$$

APPLICATIONS

Problem 7.20 *[7.73]* Figure 7.18 shows a circuit for setting the length of time voltage is applied to the electrodes of a welding machine. The time is taken as how long it takes the capacitor to charge from 0 to 8 V. What is the time range covered by the variable resistor?

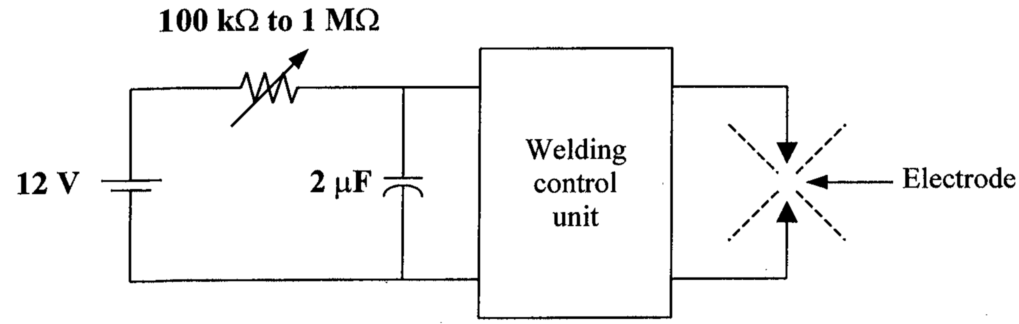


Figure 7.18

- **Carefully DEFINE the problem.**
  Each component is labeled completely. The problem is clear.

- **PRESENT everything you know about the problem.**
  This is an RC circuit with a dc voltage source. When the welding machine is activated, the dc source supplies power to the RC circuit. To find the time in which it takes the capacitor to charge from 0 to 8 V, we need to find the response of the RC circuit, written as

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$$

  where $v(0)$ is the initial voltage across the capacitor, $v(\infty)$ is the steady-state value of the voltage across the capacitor, and τ is the time constant of the RC circuit.

- **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**
  The three solution techniques that can be used are nodal analysis, mesh analysis, and basic circuit analysis. Basic circuit analysis can be used to solve this problem.

- **ATTEMPT a problem solution.**

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty) \right] e^{-t/\tau}$$

$$v(0) = 0 \text{ V} \qquad \text{and} \qquad v(\infty) = 12 \text{ V}$$

$$v(t) = (12)(1 - e^{-t/\tau})$$

  Let the voltage at an unknown time, t_0, be equal to 8 V.

$$v(t_0) = 8 = (12)(1 - e^{-t_0/\tau})$$

$$\frac{8}{12} = 1 - e^{-t_0/\tau} \longrightarrow e^{-t_0/\tau} = \frac{1}{3}$$

$$t_0 = \tau \ln(3)$$

For $R = 100\ k\Omega$,

$$\tau = RC = (100 \times 10^3)(2 \times 10^{-6}) = 0.2\ s$$

$$t_0 = 0.2 \ln(3) = 0.2197\ s$$

For $R = 1\ M\Omega$,

$$\tau = RC = (1 \times 10^6)(2 \times 10^{-6}) = 2\ s$$

$$t_0 = 2 \ln(3) = 2.197\ s$$

Thus,

$$0.2197\ s < t_0 < 2.197\ s$$

- **EVALUATE the solution and check for accuracy.**

For $R = 100\ k\Omega$,

$$\tau = RC = (100 \times 10^3)(2 \times 10^{-6}) = 0.2\ s$$

$$v(0.2197) = (12)(1 - e^{-t/\tau}) = (12)(1 - e^{-0.2197/0.2}) = 8\ V$$

For $R = 1\ M\Omega$,

$$\tau = RC = (1 \times 10^6)(2 \times 10^{-6}) = 2\ s$$

$$v(2.197) = (12)(1 - e^{-t/\tau}) = (12)(1 - e^{-2.197/2}) = 8\ V$$

Our check for accuracy was successful.

- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

$$\underline{\mathbf{0.2197\ s < t_0 < 2.197\ s}}$$

CHAPTER 8 - SECOND-ORDER CIRCUITS

List of topics for this chapter :

- Finding Initial Values
- The Source-Free Series RLC Circuit
- The Source-Free Parallel RLC Circuit
- Step Response of a Series RLC Circuit
- Step Response of a Parallel RLC Circuit
- General Second-Order Circuits
- Second-Order Op Amp Circuits

FINDING INITIAL VALUES

Problem 8.1 Given the circuit shown in Figure 8.1, which has existed for a long time, find $v_{C1}(0)$, $v_{C2}(0)$, $i_{L1}(0)$, and $i_{L2}(0)$.

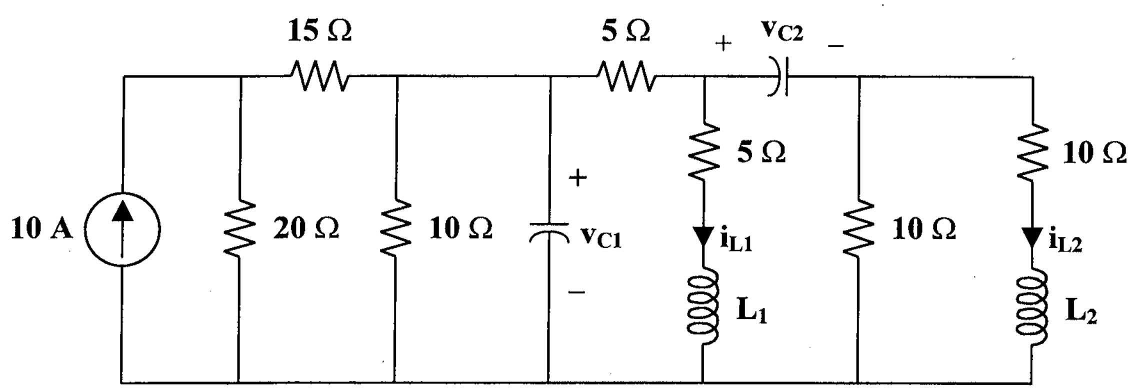
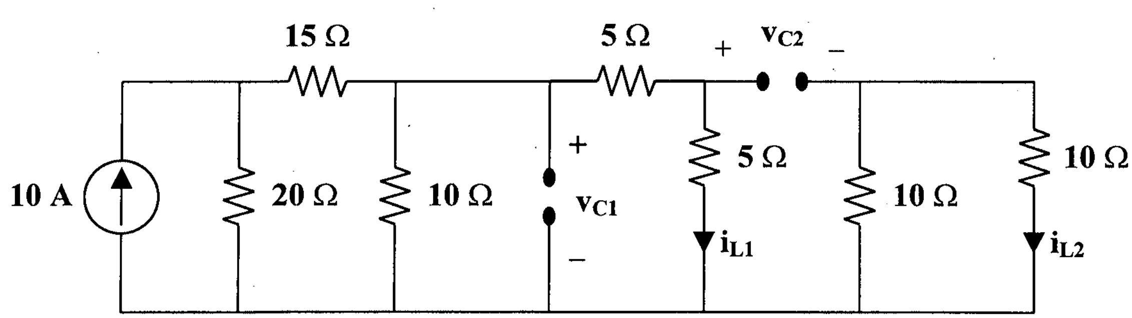
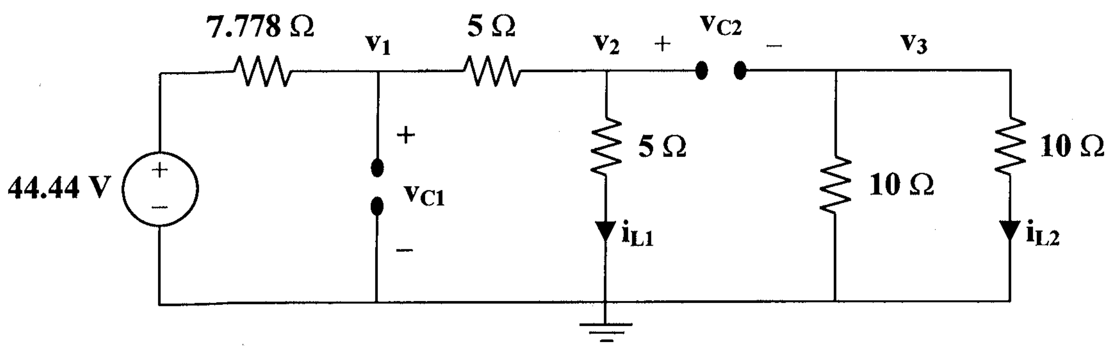


Figure 8.1

When a circuit reaches steady state, an inductor looks like a short circuit and a capacitor looks like an open circuit. So, use the following circuit to find the initial values.



Use source transformations to simplify the circuit.



Now, it is evident that

$$v_{C1}(0) = v_1 \qquad i_{L1}(0) = \frac{v_2}{5}$$

$$v_{C2}(0) = v_2 - v_3 \qquad i_{L2}(0) = \frac{v_3}{10}$$

Use nodal analysis to find v_1, v_2, and v_3.

At node 1 :
$$\frac{v_1 - 44.44}{7.778} + \frac{v_1 - v_2}{5} = 0$$
$$5(v_1 - 44.44) + 7.778(v_1 - v_2) = 0$$
$$12.778\,v_1 - 7.778\,v_2 = 222.2$$

At node 2 :
$$\frac{v_2 - v_1}{5} + \frac{v_2}{5} = 0$$
$$-v_1 + 2v_2 = 0$$
$$v_2 = \frac{v_1}{2}$$

At node 3 :
$$\frac{v_3}{10} + \frac{v_3}{10} = 0$$
$$2v_3 = 0$$
$$v_3 = 0 \text{ volts}$$

Substitute the equation from node 2 into the equation for node 1.

$$12.778\,v_1 - 7.778\frac{v_1}{2} = 222.2$$
$$8.889\,v_1 = 222.2$$
$$v_1 = 25 \text{ volts}$$

Then, $$v_2 = \frac{v_1}{2} = \frac{25}{2} = 12.5 \text{ volts}$$

Therefore,

$v_{C1}(0) = \underline{\mathbf{25\ volts}}$ $i_{L1}(0) = \underline{\mathbf{2.5\ amps}}$

$v_{C2}(0) = \underline{\mathbf{12.5\ volts}}$ $i_{L2}(0) = \underline{\mathbf{0\ amps}}$

Problem 8.2 Given the circuit shown in Figure 8.2, which has existed for a long time, find $i_L(0)$ and $v_C(0)$.

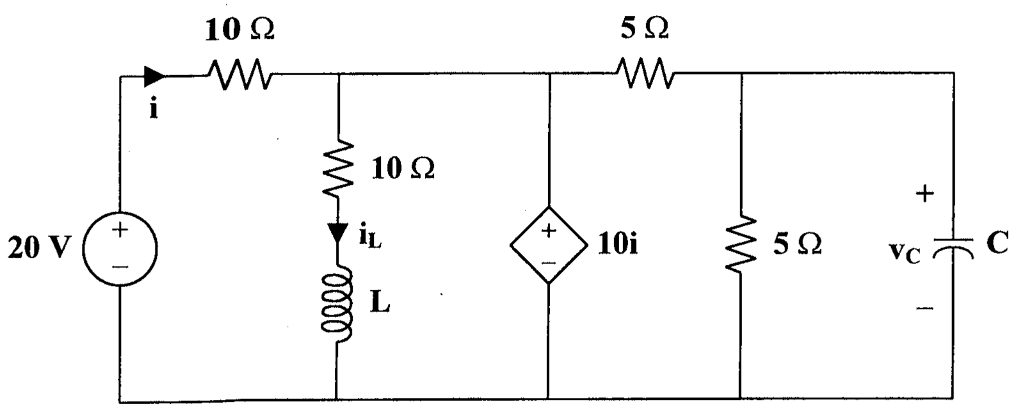


Figure 8.2

$i_L(0) = \underline{\mathbf{1\ amp}}$ $v_C(0) = \underline{\mathbf{5\ volts}}$

THE SOURCE-FREE SERIES RLC CIRCUIT

Problem 8.3 Given the circuit in Figure 8.3, which has reached steady state before the switch closes, find $i(t)$ for all $t > 0$.

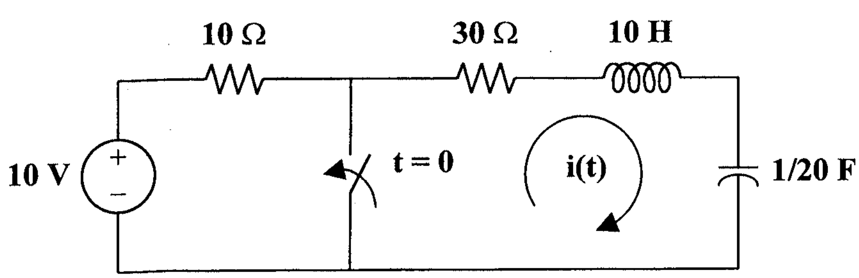


Figure 8.3

Use KVL to write a loop equation for $t > 0$.

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt = 0$$

Multiply by $1/L$ and differentiate with respect to time.

$$\frac{R}{L}\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} + \frac{1}{LC}i(t) = 0$$

Rearranging the terms and inserting the values for R, L, and C,

$$\frac{d^2i(t)}{dt^2} + 3\frac{di(t)}{dt} + 2i(t) = 0$$

Assume a solution of Ae^{st}.

$$s^2Ae^{st} + 3sAe^{st} + 2Ae^{st} = 0$$
$$(s^2 + 3s + 2)\,Ae^{st} = 0$$

Thus,

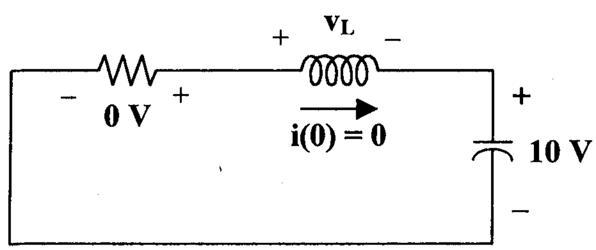
$$(s+1)(s+2) = 0$$

which gives real and unequal roots at $s_1 = -1$ and $s_2 = -2$.

Hence,

$$i(t) = A_1e^{-t} + A_2e^{-2t}$$

At $t = 0^+$, the circuit is



So,

$$i(0) = 0 = A_1 + A_2 \qquad \text{or} \qquad A_2 = -A_1$$

Also,

$$v_L(0^+) = 10\frac{di(0)}{dt} = -10 \text{ volts} \qquad \text{or} \qquad \frac{di(0)}{dt} = -1$$

and

$$\frac{di(0)}{dt} = -A_1e^0 - 2A_2e^0 = -A_1 - 2A_2$$

So,

$$-1 = -A_1 - 2A_2 = -A_1 + 2A_1 = A_1$$

Hence,

$$A_1 = -1 \qquad \text{and} \qquad A_2 = 1$$

Therefore,

$$\underline{i(t) = \left(-e^{-t} + e^{-2t}\right) \textbf{ amps } \forall\, t > 0}$$

Problem 8.4 Given the circuit in Figure 8.4, which has reached steady state before the switch closes, find $i(t)$ for all $t > 0$.

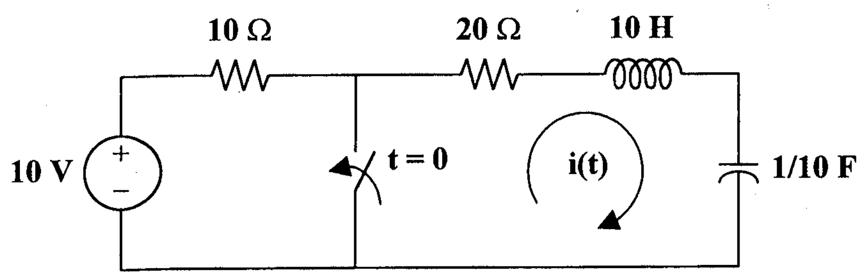


Figure 8.4

At $t = 0^-$,

$$i(0^-) = i(0^+) = 0 \text{ amps} \qquad \text{and} \qquad v_C(0^-) = v_C(0^+) = 10 \text{ volts}$$

For $t > 0$,

$$20\,i(t) + 10\frac{di(t)}{dt} + \frac{1}{1/10}\int i(t)\,dt = 0$$

Multiply by $1/10$, differentiate with respect to time, and rearrange the terms.

$$\frac{d^2i(t)}{dt^2} + 2\frac{di(t)}{dt} + i(t) = 0$$

Again, using a solution of Ae^{st},

$$s^2Ae^{st} + 2sAe^{st} + Ae^{st} = 0$$
$$(s^2 + 2s + 1)Ae^{st} = 0$$

Thus,

$$(s+1)^2 = 0$$

which gives a real and repeated root at $s_{1,2} = -1$.

A repeated root gives the following solution,

$$i(t) = A_1e^{-t} + A_2te^{-t}$$

At $t = 0$,

$$i(0) = 0 = A_1e^0 + A_2(0)e^0 = A_1 \qquad \text{or} \qquad A_1 = 0$$

Also,

$$v_L(0) = 10\frac{di(0)}{dt} = \text{-}10 \qquad \text{or} \qquad \frac{di(0)}{dt} = \text{-}1$$

and
$$\frac{di(0)}{dt} = 0 + A_2 e^0 - A_2(0)e^0 = A_2$$

Hence,
$$A_2 = -1$$

Therefore,
$$\underline{i(t) = \left(\text{-}\,\mathbf{te^{-t}}\right)\ \mathbf{amps}\ \forall\ \mathbf{t > 0}}$$

Problem 8.5 Given the circuit in Figure 8.5, which has reached steady state before the switch closes, find $i(t)$ for all $t > 0$.

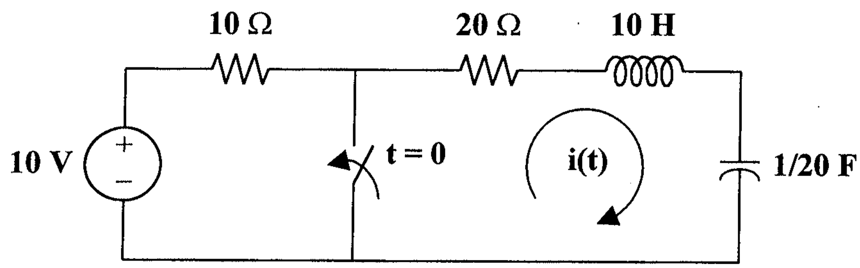


Figure 8.5

Writing a loop equation for $t > 0$ gives,

$$20i(t) + 10\frac{di(t)}{dt} + \frac{1}{1/20}\int i(t)\,dt = 0$$

Multiply by $1/10$, differentiate with respect to time, and rearrange the terms.

$$\frac{d^2i(t)}{dt^2} + 2\frac{di(t)}{dt} + 2i(t) = 0$$

Again, using a solution of Ae^{st},

$$s^2Ae^{st} + 2sAe^{st} + 2Ae^{st} = 0$$
$$(s^2 + 2s + 2)\,Ae^{st} = 0$$

Thus,
$$(s + 1 + j)(s + 1 - j) = 0$$
which gives complex roots at $s_{1,2} = \text{-}1 \mp j$.

Hence, we have a solution

$$i(t) = A_1 e^{(\text{-}1\text{-}j)t} + A_2 e^{(\text{-}1+j)t}$$

At $t = 0$,

$$i(0) = 0 = A_1 e^0 + A_2 e^0 = A_1 + A_2 \quad \text{or} \quad A_2 = -A_1$$

Also,

$$\frac{di(0)}{dt} = -1 = (-1 - j)A_1 e^0 + (-1 + j)A_2 e^0$$

Using $A_2 = -A_1$, we get

$$-1 = (-1 - j)\, A_1 + (-1 + j)(-A_1)$$
$$-1 = (-1 - j + 1 - j)A_1 = -2jA_1$$

or $\quad A_1 = \dfrac{1}{2j} \quad$ and $\quad A_2 = \dfrac{-1}{2j}$

Therefore,

$$i(t) = \frac{1}{2j} e^{(-1-j)\,t} + \frac{-1}{2j} e^{(-1+j)\,t}$$

$$i(t) = -e^{-t}\left\{\frac{e^{jt} - e^{-jt}}{2j}\right\}$$

$$i(t) = \underline{\left(-\mathbf{e^{-t}\,sin(t)}\right)\ \textbf{amps}\ \forall\ \mathbf{t > 0}}$$

THE SOURCE-FREE PARALLEL RLC CIRCUIT

Problem 8.6 Given the circuit in Figure 8.6, find $v_C(t)$ for all $t > 0$.

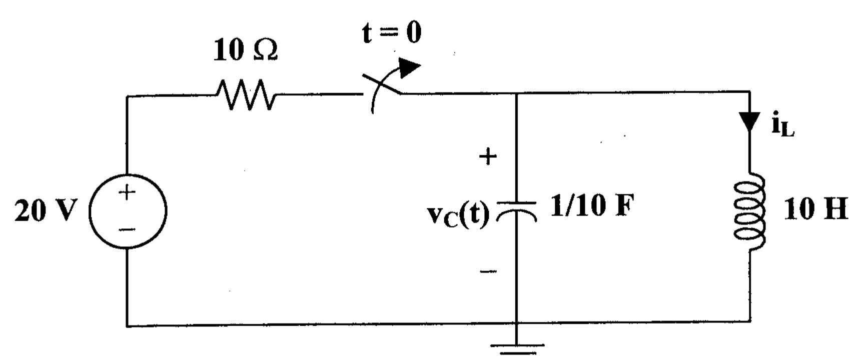


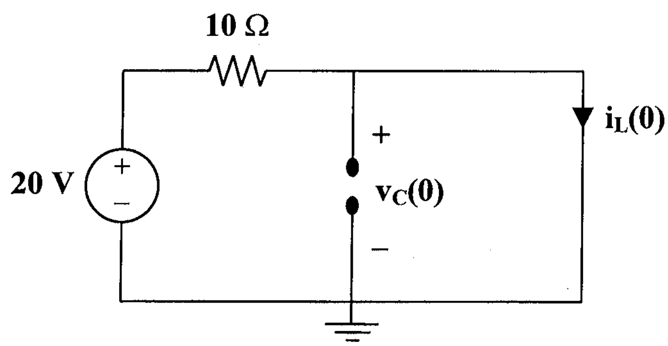
Figure 8.6

- **Carefully DEFINE the problem.**
  Each component is labeled, indicating the value and polarity. The problem is clear.

➢ **PRESENT everything you know about the problem.**

The goal of the problem is to find $v_C(t)$ for all $t > 0$.

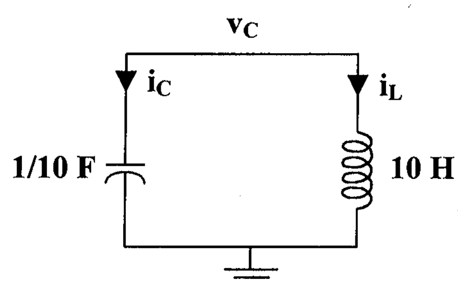
There is a switch which opens at $t = 0$. So, there are two circuits. The first circuit, when the switch is closed, is used to find the initial values of the capacitor and inductor. Note that there is a dc source. At dc, a capacitor is an open circuit and an inductor is a short circuit. Thus, we have the following circuit.



Recall that the voltage of a capacitor cannot change instantaneously and the current through an inductor cannot change instantaneously.

$$v_C(0) = v_C(0^-) = v_C(0^+) \qquad \text{and} \qquad i_L(0) = i_L(0^-) = i_L(0^+)$$

The second circuit, after the switch opens, is used to find the final solution.



➢ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**

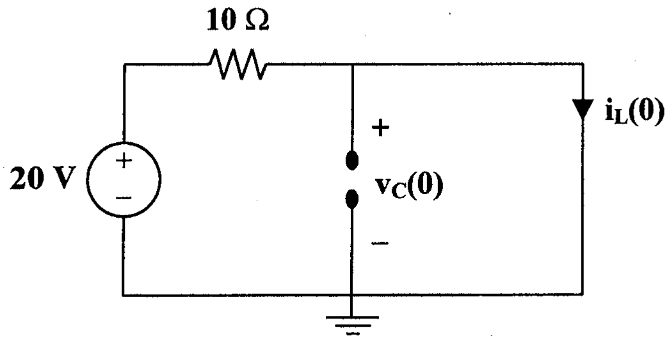
The first circuit was simplified by applying the characteristics of capacitors and inductors with a dc source. Ohm's law should provide the answer you need when the circuit consists of a dc voltage source and a resistor.

For the second circuit, there is only one node or one loop. In this case, the use of KCL or KVL should provide the desired equation to find the solution to the problem. Because the components are in parallel, the voltage across each component is the same. So, use KCL to find the currents in terms of the voltage v_C.

➢ **ATTEMPT a problem solution.**

Begin by finding the initial values of the capacitor and inductor.

At $t = 0$,



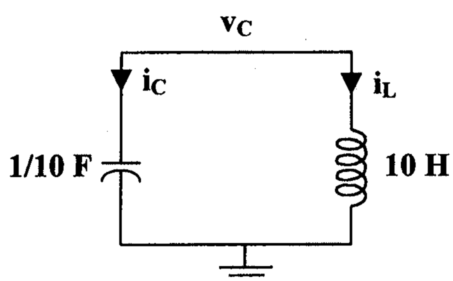
It is evident from the circuit that

$$v_C(0) = 0 \text{ volts.}$$

Using Ohm's law,

$$i_L(0) = 20/10 = 2 \text{ amps}$$

After the switch opens, the circuit becomes



Using KCL,

$$i_C + i_L = 0$$

$$\frac{1}{10}\frac{d(v_C - 0)}{dt} + \frac{1}{10}\int (v_C - 0)\, dt = 0$$

Multiply both sides of the equation by 10 and differentiate both sides with respect to time.

$$\frac{d^2 v_C}{dt^2} + v_C = 0$$

but Ae^{st} must be a solution.

Substituting,

$$\frac{d^2\left(Ae^{st}\right)}{dt^2} + Ae^{st} = 0$$

$$s^2 Ae^{st} + Ae^{st} = 0$$

$$(s^2 + 1)Ae^{st} = 0$$

Thus,

$$(s^2+1) = (s-j)(s+j) = 0$$

which gives complex roots at $s_1, s_2 = \pm j$

So, we have a solution

$$v_C(t) = A_1e^{jt} + A_2e^{-jt}$$

Now, to solve for A_1 and A_2.

$$v_C(0) = A_1 + A_2 = 0 \qquad \text{or} \qquad A_2 = -A_1$$

Now,

$$v_C(t) = A_1e^{jt} - A_1e^{-jt}$$

$$i_C(0) = -i_L(0) = -2 \text{ amps}$$

$$i_C(0) = \frac{1}{10}\frac{dv_C(0)}{dt} = \frac{1}{10}(jA_1e^0 + jA_1e^0)$$

or

$$\frac{j2A_1}{10} = -2$$

Hence,

$$A_1 = \frac{-20}{j2} = \frac{-10}{j} = j10$$

Therefore,

$$v_C(t) = j10e^{jt} - j10e^{-jt}$$

$$v_C(t) = 10\left[e^{j90°}e^{jt} + e^{-j90°}e^{-jt}\right]$$

$$v_C(t) = 10\left[e^{j(t+90°)} + e^{-j(t+90°)}\right]$$

$$v_C(t) = 10\left[2\cos(t+90°)\right]$$

$$v_C(t) = 20\cos(t+90°) \text{ volts } \forall\ t > 0$$

- **EVALUATE the solution and check for accuracy.**
  The circuit must satisfy the conservation of energy. In this case, the energy supplied from one component is absorbed by the other component.

  Recall, from Chapter 6 – Capacitors and Inductors, that the energy stored in a capacitor is

$$w = \frac{1}{2}Cv^2$$

and the energy stored in an inductor is

$$w = \frac{1}{2}Li^2$$

Thus,

$$w = \frac{1}{2}Cv^2 = \left(\frac{1}{2}\right)\left(\frac{1}{10}\right)(20)^2 = 20 \text{ joules}$$

and

$$w = \frac{1}{2}Li^2 = \left(\frac{1}{2}\right)(10)(2)^2 = 20 \text{ joules}$$

These two answers match. This circuit satisfies conservation of energy.

➤ **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
This problem has been solved satisfactorily.

$$v_C(t) = \underline{\mathbf{20\cos(t+90°)\ volts\ \forall\ t>0}}$$

Problem 8.7 *[8.23]* In the circuit in Figure 8.7, calculate $i_o(t)$ and $v_o(t)$ for $t > 0$.

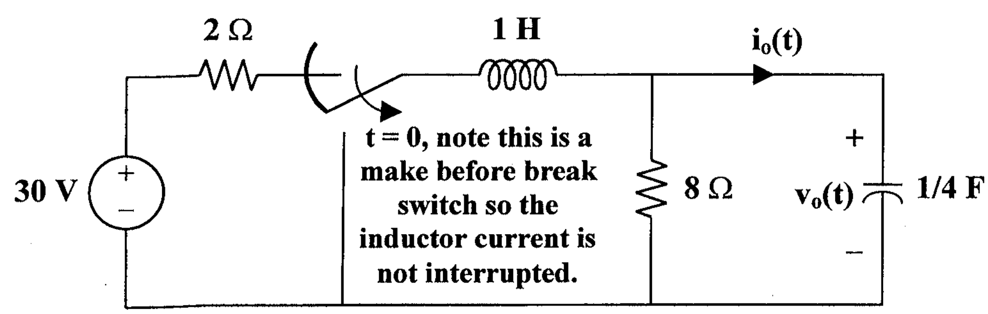
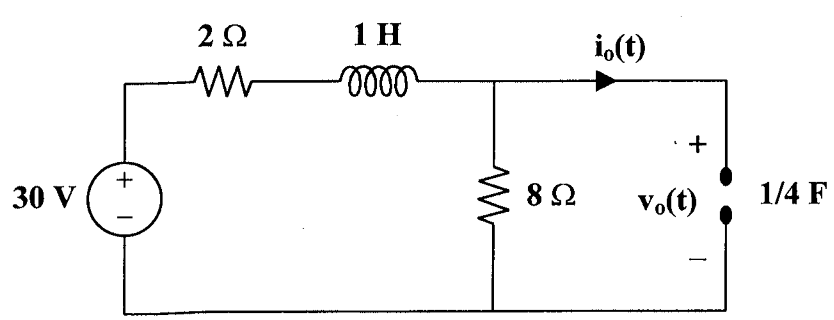


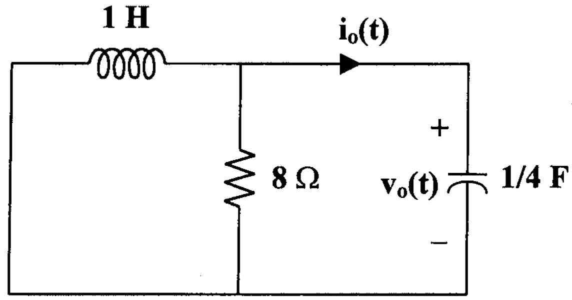
Figure 8.7

At $t = 0$,



$$v_o(0) = \frac{8}{2+8}(30) = 24 \qquad \text{and} \qquad i_o(0) = 0$$

For $t > 0$, we have a source-free parallel RLC circuit.



$$\alpha = \frac{1}{2RC} = \frac{1}{(2)(8)(1/4)} = \frac{1}{4}$$

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1)(1/4)}} = 2$$

Since α is less than ω_o, we have an underdamped response with a damping frequency of

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{4-(1/16)} = 1.9843$$

and the natural response is

$$v_o(t) = (A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t))e^{-\alpha t}$$

Use the initial values to find the unknowns.

$$v_o(0) = 24 = A_1$$

Also,

$$i_o(0) = C\frac{dv_o(0)}{dt} = 0$$

where $\dfrac{dv_o}{dt} = -\alpha(A_1 \cos(\omega_d t) + A_2 \sin(\omega_d t))e^{-\alpha t} + (-\omega_d A_1 \sin(\omega_d t) + \omega_d A_2 \cos(\omega_d t))e^{-\alpha t}$

So,

$$\frac{dv_o(0)}{dt} = 0 = -\alpha A_1 + \omega_d A_2$$

Thus,

$$A_2 = \frac{\alpha}{\omega_d}A_1 = \frac{(1/4)(24)}{1.9843} = 3.024$$

Therefore,

$$v_o(t) = \underline{\left[(\mathbf{24\cos(\omega_d t) + 3.024\sin(\omega_d t)})\mathbf{e^{-t/4}}\right] \textbf{volts } \forall\ \mathbf{t > 0}}$$

Problem 8.8 Given the circuit in Figure 8.8, which has reached steady state before the switch closes, find $v(t)$ for all $t > 0$.

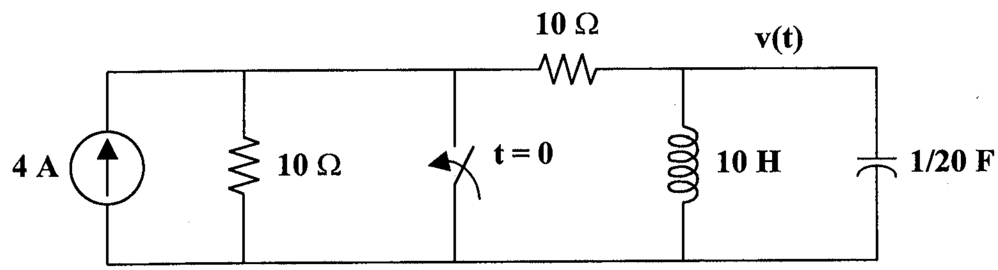


Figure 8.8

$$v(t) = \underline{\mathbf{40\,e^{-t}\cos(t + 90°)\ volts\ \forall\ t > 0}}$$

STEP RESPONSE OF A SERIES RLC CIRCUIT

Problem 8.9 *[8.25]* A branch voltage in a series RLC circuit is described by

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 8v = 24$$

If the initial conditions are $v(0) = 0 = \dfrac{dv(0)}{dt}$, find $v(t)$.

Recall the general loop equation for a series RLC circuit.

$$L\frac{di}{dt} + Ri + v = V_S$$

where $i = C\,dv/dt$. Then,

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_S}{LC}$$

The complete response of this circuit has a forced response and a natural response. To find the forced response, realize that $8V_S = 24$ which means that $V_S = 3$. This is the forced response. Now, to find the natural response, let $V_S = 0$ and

$$\frac{d^2v}{dt^2} + 4\frac{dv}{dt} + 8v = 0$$

$$(s^2 + 4s + 8)v = 0$$

Thus,

$$s^2 + 4s + 8 = 0$$

which gives complex roots at $s_{1,2} = \dfrac{-4 \pm \sqrt{16-32}}{2} = -2 \pm j2$.

So, we have the solution

$$v(t) = V_S + (A_1 \cos(2t) + A_2 \sin(2t))e^{-2t}$$

Solve for the unknowns.

$$v(0) = 0 = V_s + A_1 = 3 + A_1 \qquad \text{or} \qquad A_1 = -3$$

$$\frac{dv}{dt} = -2(A_1 \cos(2t) + A_2 \sin(2t))e^{-2t} + (-2A_1 \sin(2t) + 2A_2 \cos(2t))e^{-2t}$$

$$\frac{dv(0)}{dt} = 0 = -2A_1 + 2A_2 \qquad \text{or} \qquad A_2 = A_1 = -3$$

Therefore,

$$v(t) = \underline{\mathbf{[3 - 3(\cos(2t) + \sin(2t))e^{-2t}]\ volts}}$$

Problem 8.10 *[8.31]* Calculate $i(t)$ for $t > 0$ using the circuit in Figure 8.9.

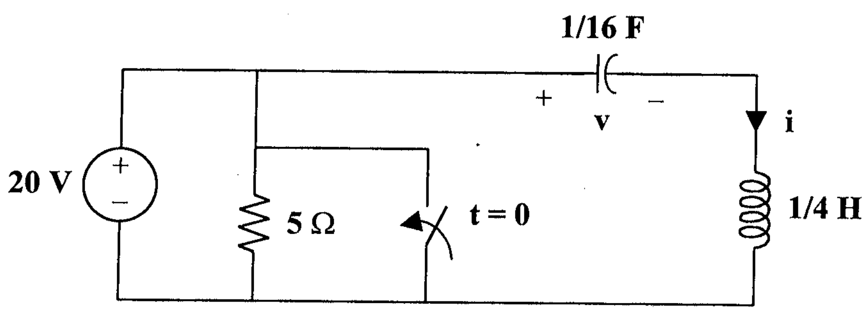
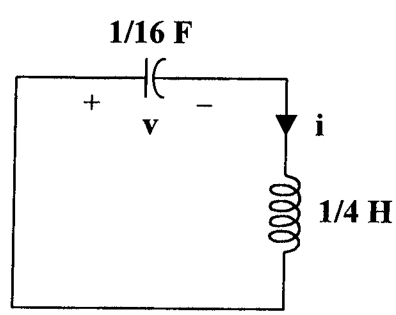


Figure 8.9

Before $t = 0$, the capacitor acts like an open circuit while the inductor acts like a short circuit.

$i(0) = 0$ amps and $v(0) = 20$ volts

For $t > 0$, the LC circuit is disconnected from the voltage source as shown below.



This is a lossless, source-free series RLC circuit.

$$\alpha = \frac{R}{2L} = 0 \qquad \omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1/16)+(1/4)}} = 8$$

$$s = -\alpha \pm j\omega_d = -\alpha \pm j\sqrt{\omega_o^2 - \alpha^2} = \pm j\omega_o = \pm j8$$

Since α is equal to zero, we have an undamped response. Hence,

$$i(t) = A_1 \cos(8t) + A_2 \sin(8t)$$

where $i(0) = 0 = A_1$.

So,

$$i(t) = A_2 \sin(8t)$$

To solve for A_2, we know that

$$\frac{di(0)}{dt} = \frac{1}{L} v_L(0) = \frac{-1}{L} v(0) = (-4)(20) = -80$$

However,

$$\frac{di}{dt} = 8 A_2 \cos(8t)$$

and $$\frac{di(0)}{dt} = -80 = 8 A_2$$

which leads to $A_2 = -10$.

Therefore,

$$i(t) = \underline{\mathbf{-10 \sin(8t)\ amps}\ \forall\ \mathbf{t > 0}}$$

STEP RESPONSE OF A PARALLEL RLC CIRCUIT

Problem 8.11 Given the circuit in Figure 8.10, find $v(t)$ for all $t > 0$.

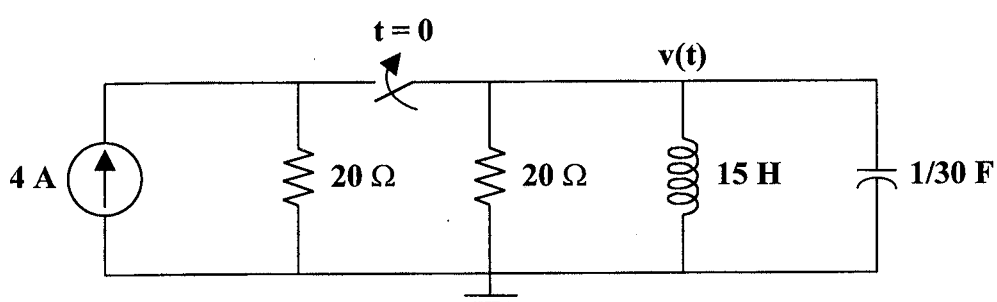


Figure 8.10

$$v(t) = \underline{\left(120\,e^{-t} - 120\,e^{-2t}\right) \text{ volts } \forall\ t > 0}$$

Problem 8.12 Given the circuit in Figure 8.11, find $v(t)$ for all $t > 0$.

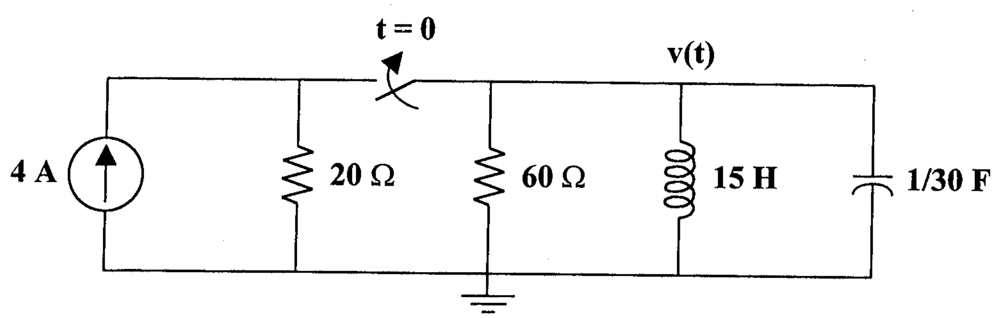


Figure 8.11

At $t = 0^-$,

$$v_C(0) = 0 \text{ volts and } i_L(0) = 0 \text{ amps.}$$

At $t = 0^+$,

$$v_C = v_L = v_R = 0 \text{ volts}$$

So, the 4 amp current source must all go through the capacitor or

$$i_C(0^+) = 4 \text{ amps.}$$

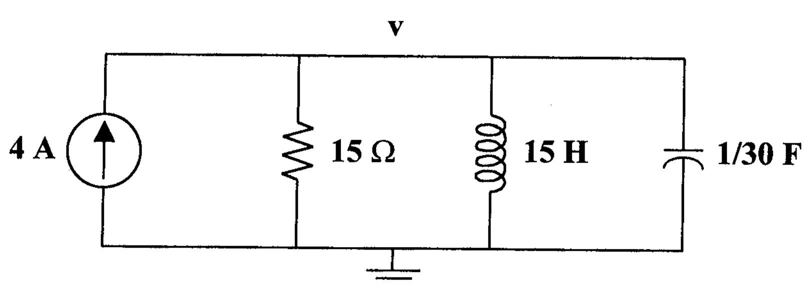
Hence,

$$C\frac{dv(0^+)}{dt} = i_C(0^+) = 4$$

or

$$\frac{dv(0^+)}{dt} = \frac{4}{C} = \frac{4}{1/30} = 120 \text{ volts/sec}$$

For $t > 0$, the circuit becomes



where the 20 ohm resistor in parallel with the 60 ohm resistor is equivalent to a 15 ohm resistor.

Using nodal analysis,

$$-4+\frac{v-0}{15}+\frac{1}{15}\int(v-0)\,dt+\frac{1}{30}\frac{d(v-0)}{dt}=0$$

Multiplying by 30 and differentiating with respect to time yields

$$\frac{d^2v}{dt^2}+2\frac{dv}{dt}+2v=0$$

Substituting the solution of Ae^{st} produces

$$s^2Ae^{st}+2sAe^{st}+2Ae^{st}=0$$
$$(s^2+2s+2)Ae^{st}=0$$

Hence,

$$s^2+2s+2=(s+1+j)(s+1-j)=0$$

which gives complex roots at $s_{1,2}=-1\mp j$.

So, we have the solution

$$v(t)=A_1e^{-(1+j)t}+A_2e^{-(1-j)t}$$

At $t=0$,

$$v(0)=A_1\,e^0+A_2\,e^0=A_1+A_2=0 \qquad \text{or} \qquad A_2=-A_1$$

Also,

$$\frac{dv(0)}{dt}=\left[-(1+j)\right]A_1-\left[-(1-j)\right]A_1=120$$
$$-A_1-jA_1+A_1-jA_1=-2jA_1=120$$
$$A_1=\frac{120}{-2j}=60j=60e^{j} \qquad \text{and} \qquad A_2=-A_1=60e^{-j}$$

Therefore,

$$\begin{aligned} v(t) &= 60e^{j}e^{-(1+j)t}+60e^{-j}e^{-(1-j)t} \\ &= 60e^{-t}\left\{e^{-jt+j}+e^{jt-j}\right. \\ &= 60e^{-t}\,[2\cos(t-90°)] \\ v(t) &= \underline{\mathbf{120e^{-t}\cos(t-90°)\ volts\ \forall\ t>0}} \end{aligned}$$

The answer can also be written as

$$v(t)=\underline{\mathbf{120e^{-t}\sin(t)\ volts\ \forall\ t>0}}$$

Problem 8.13 Given the circuit in Figure 8.12, find $v(t)$ for all $t > 0$.

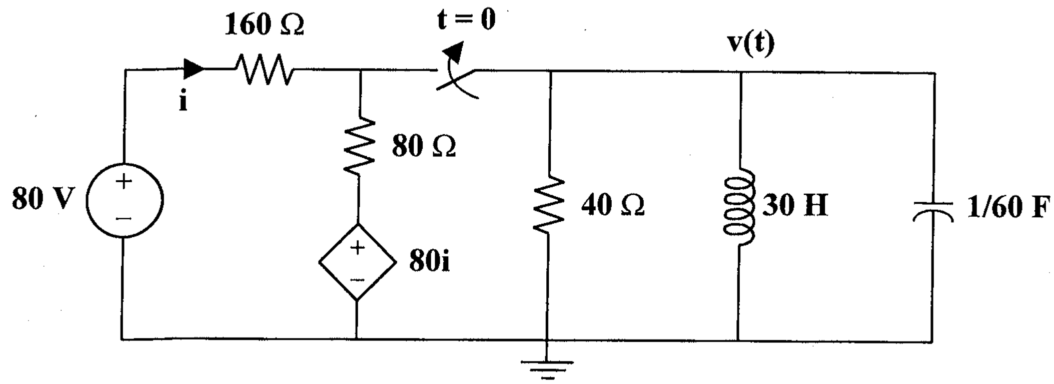


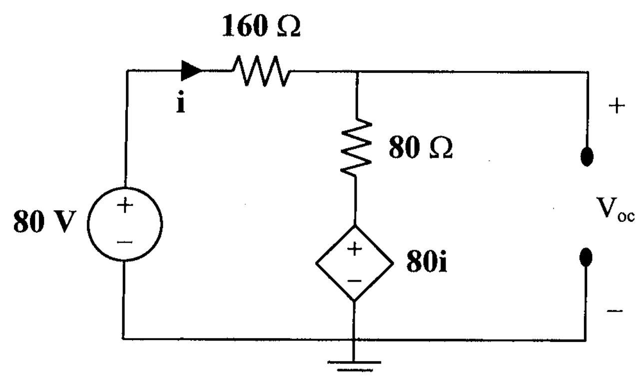
Figure 8.12

At $t = 0$,

$$i_L(0) = 0 \text{ amps} \qquad \text{and} \qquad v_C(0) = 0 \text{ volts}$$

For complicated circuits that can be simplified using either a Thevenin or Norton equivalent, do so immediately!

Solving for V_{oc},



Using nodal analysis,

$$\frac{V_{oc} - 80}{160} + \frac{V_{oc} - 80i}{80} = 0$$

$$(V_{oc} - 80) + 2(V_{oc} - 80i) = 0$$

$$3V_{oc} - 80 - 160i = 0$$

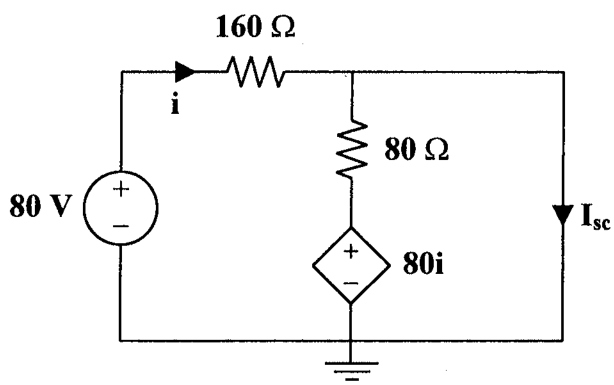
where $i = \dfrac{80 - V_{oc}}{160}$.

Hence,

$$3V_{oc} = 80 + 160\left(\frac{80 - V_{oc}}{160}\right)$$

$$3V_{oc} = 80 + (80 - V_{oc})$$
$$4V_{oc} = 160$$
$$V_{oc} = 40 \text{ volts}$$

Solving for I_{sc},



Using KCL,

$$I_{sc} = \frac{80-0}{160} + \frac{80i-0}{80}$$

where $i = \dfrac{80-0}{160} = \dfrac{1}{2}$ amp

Hence,

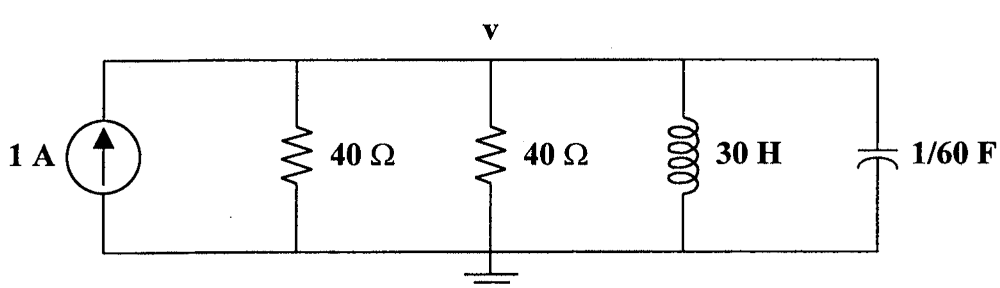
$$I_{sc} = \frac{80}{160} + \frac{80i}{80} = \frac{1}{2} + i = \frac{1}{2} + \frac{1}{2} = 1 \text{ amp}$$

Finding a Norton equivalent circuit,

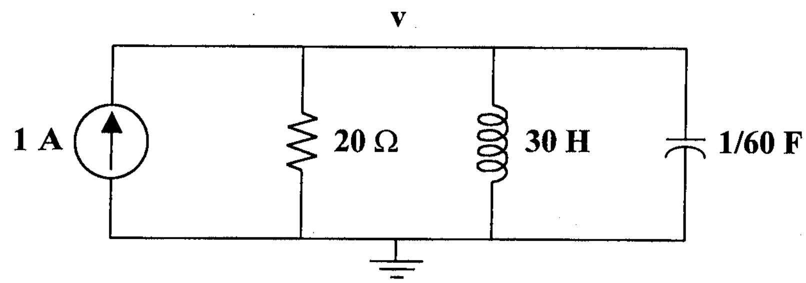
$$I_N = I_{sc} = 1 \text{ amp}$$

and $$R_{eq} = R_N = \frac{V_{oc}}{I_{sc}} = \frac{40}{1} = 40 \text{ ohms}$$

Now, we can work with the following simplified circuit,



The two 40 ohm resistors in parallel convert to a 20 ohm resistor, so we can simplify the circuit further to become



Writing a single node equation results in

$$-1+\frac{v-0}{20}+\frac{1}{30}\int(v-0)\,dt+\frac{1}{60}\frac{d(v-0)}{dt}=0$$

Multiplying by 60 and differentiating with respect to time leads to

$$\frac{d^2v}{dt^2}+3\frac{dv}{dt}+2v=0$$

Substituting the solution Ae^{st}, we get

$$s^2Ae^{st}+3sAe^{st}+2Ae^{st}=0$$
$$(s^2+3s+2)Ae^{st}=0$$

Hence,

$$s^2+3s+2=(s+1)(s+2)=0$$

which gives real and unequal roots at $s_1 = -1$ and $s_2 = -2$.

So, we have the solution

$$v(t)=A_1e^{-t}+A_2e^{-2t}$$

At $t=0$,

$v(0)=0$ volts and $i_L(0)=0$ amps.

Since $v(0)=0$ volts,

$v_R=0$ volts and $i_R=0$ amps.

Hence, the entire 1 amp of current flows through the capacitor. So, we have

$i_C(0)=1$ amp and $i_C(0)=C\dfrac{dv(0)}{dt}$

Thus,

$$\frac{dv(0)}{dt}=\frac{1}{C}i_C(0)=\frac{1}{1/60}(1)=60 \text{ volts/sec}$$

but $\dfrac{dv(0)}{dt}=-A_1e^0-2A_2e^0=-A_1-2A_2=60$

Also,

$$v(0) = A_1 e^0 + A_2 e^0 = A_1 + A_2 = 0$$

implies that $A_2 = -A_1$.

So,

$$-A_1 - 2A_2 = 60$$

or

$$-A_1 + 2A_1 = A_1 = 60$$

Hence,

$$A_1 = 60 \qquad \text{and} \qquad A_2 = -60.$$

Therefore,

$$\underline{v(t) = \left(\mathbf{60\,e^{-t} - 60\,e^{-2t}}\right) \textbf{volts} \ \forall\ \mathbf{t > 0}}$$

The power of using a Norton equivalent circuit is clearly demonstrated by this problem.

GENERAL SECOND-ORDER CIRCUITS

Problem 8.14 Given the circuit in Figure 8.13, find $v_C(t)$ for all $t > 0$.

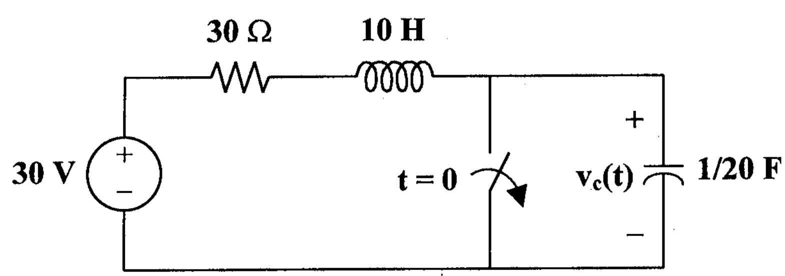


Figure 8.13

This is a second-order circuit with a forced response and a natural response.

Solve for initial conditions.
At $t = 0$,

$$v_C(0^-) = v_C(0^+) = 0 \quad \text{and} \quad i_L(0^-) = i_L(0^+) = \frac{30}{30} = 1 \text{ amp.}$$

Hence,

$$i_C(0^+) = C\frac{dv_C(0^+)}{dt} = 1 \text{ amp}$$

or $$\frac{dv_C(0^+)}{dt} = \frac{1}{C}i_C(0^+) = \frac{1}{1/20}(1) = 20 \text{ volts/sec}$$

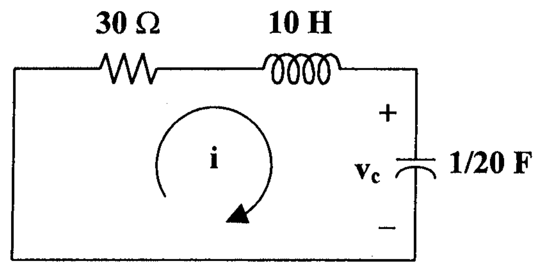
Solving for final values,

$$i_C(\infty) = 0 \text{ amps} \qquad \text{and} \qquad v_C(\infty) = 30 \text{ volts}$$

which is also the forced response,

$$v_{C_f} = 30 \text{ volts.}$$

Solving for the natural response,



The loop equation is

$$Ri + L\frac{di}{dt} + v_C = 0$$

where $i = C\dfrac{dv_C}{dt}$.

So, we now have,

$$RC\frac{dv_C}{dt} + LC\frac{d^2v_C}{dt^2} + v_C = 0$$

Substituting for the values of R, L, and C

$$\frac{30}{20}\frac{dv_C}{dt} + \frac{10}{20}\frac{d^2v_C}{dt^2} + v_C = 0$$

Simplifying and rearranging the terms,

$$\frac{d^2v_C}{dt^2} + 3\frac{dv_C}{dt} + 2v_C = 0$$

Substituting a solution $v_C = Ae^{st}$ yields

$$s^2Ae^{st} + 3sAe^{st} + 2Ae^{st} = 0$$
$$(s^2 + 3s + 2)Ae^{st} = 0$$

Thus,

$$s^2 + 3s + 2 = (s+1)(s+2) = 0$$

which gives real and unequal roots at $s_1 = -1$ and $s_2 = -2$.

Hence, the natural response is

$$v_{C_n} = Ae^{-t} + Be^{-2t}$$

and the complete response is

$$v_C = v_{C_f} + v_{C_n} = 30 + Ae^{-t} + Be^{-2t}$$

Now, we need to solve for the unknowns, A and B.

$$v_C(0) = 30 + Ae^0 + Be^0 = 0$$

$$A + B = -30 \qquad \text{or} \qquad B = -A - 30$$

Also,

$$\frac{dv_C(0)}{dt} = 0 + (-A)e^0 + (-2B)e^0 = -A - 2B = 20$$

Now,

$$A + 2B = -20$$

which leads to

$$A + (2)(-A - 30) = -20$$

Hence,

$$-A - 60 = -20 \qquad \text{or} \qquad A = -40$$

Then,

$$B = -(-40) - 30 = 10$$

Therefore,

$$v_C(t) = \underline{\left(\mathbf{30 - 40\,e^{-t} + 10\,e^{-2t}}\right)\textbf{ volts } \forall\ \mathbf{t > 0}}$$

Problem 8.15 Given the circuit shown in Figure 8.14, find $i_L(t)$ for all $t > 0$.

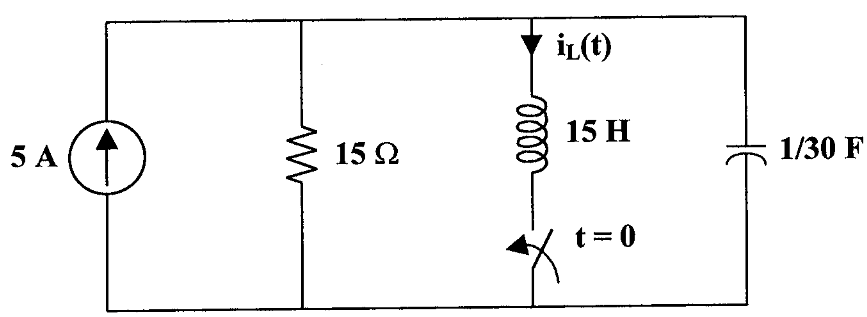


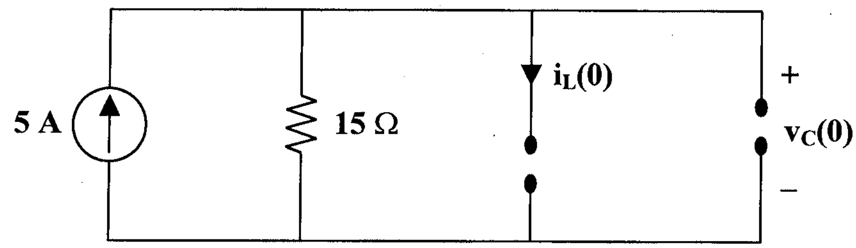
Figure 8.14

- **Carefully DEFINE the problem.**
  Each component is labeled, indicating the value and polarity. The problem is clear.

➤ **PRESENT everything you know about the problem.**

The goal of the problem is to find $i_L(t)$ for all $t > 0$.

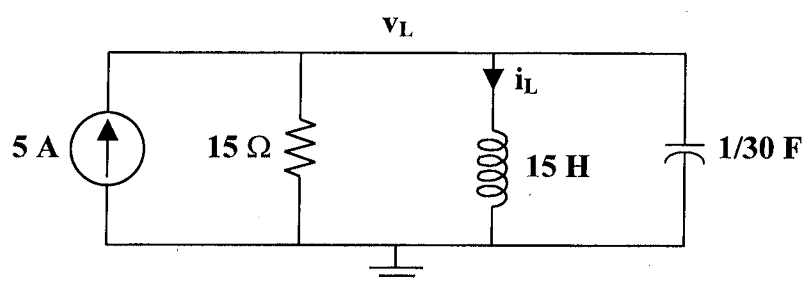
There is a switch in the circuit. So, we will need to look at two different circuits. The first circuit, when the switch is open, is used to find the initial values of the capacitor and inductor. Note that there is a dc source. At dc, a capacitor is an open circuit and an inductor is a short circuit. Thus, we have the following circuit.



Recall that the voltage of a capacitor cannot change instantaneously and the current through an inductor cannot change instantaneously.

$$v_C(0) = v_C(0^-) = v_C(0^+) \quad \text{and} \quad i_L(0) = i_L(0^-) = i_L(0^+)$$

The second circuit, after the switch opens is used to find the final solution.



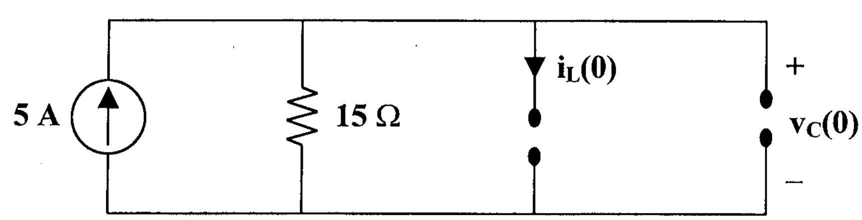
➤ **Establish a set of ALTERNATIVE solutions and determine the one that promises the greatest likelihood of success.**

This is a second-order circuit with a forced response and a natural response. The forced response is the response due to the current source at steady state. The natural response is the response of the parallel RLC circuit without the current source.

For simple resistive circuits such as the one used to find the initial values and final values, we will use observation and Ohm's law. For the parallel RLC circuit, use nodal analysis. There will be only one equation with this technique versus three equations with mesh analysis.

➤ **ATTEMPT a problem solution.**

Solve for initial conditions using the circuit below.



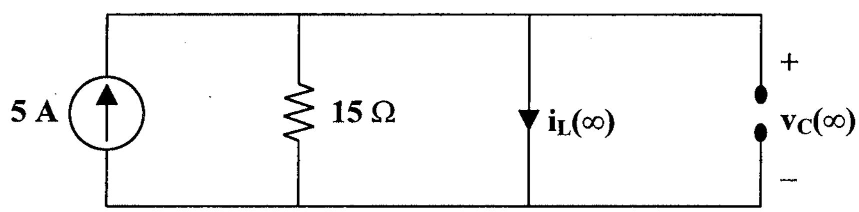
At $t = 0$,

$$v_C(0^-) = v_C(0^+) = (5)(15) = 75 \text{ volts}$$
$$i_L(0^-) = i_L(0^+) = 0$$
$$v_L(0^+) = v_C(0^+) = 75 \text{ volts}$$

Thus,

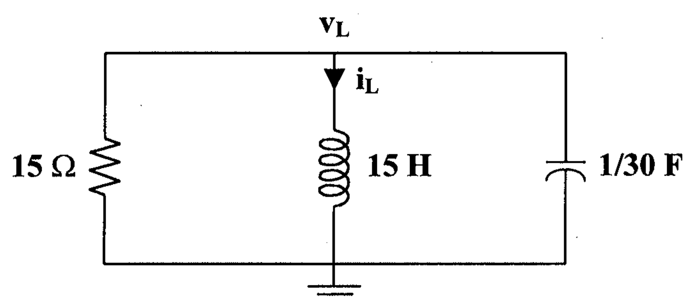
$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{75}{15} = 5 \text{ amp/sec}$$

Solving for final values, the forced response,



$$i_L(\infty) = i_{L_f} = 5 \text{ amps}$$

Now, solve for the natural response.



Writing the node equation,

$$\frac{v_L - 0}{15} + i_L + \frac{1}{30}\frac{d(v_L - 0)}{dt} = 0$$

where $v_L = 15\dfrac{di_L}{dt}$

So,

$$\frac{15}{15}\frac{di_L}{dt} + i_L + \frac{15}{30}\frac{d^2 i_L}{dt^2} = 0$$

Simplifying and rearranging the terms,

$$\frac{d^2 i_L}{dt^2} + 2\frac{di_L}{dt} + 2i_L = 0$$

Substituting the solution $i_L = Ae^{st}$ yields

$$s^2Ae^{st} + 2sAe^{st} + 2Ae^{st} = 0$$
$$(s^2 + 2s + 2)Ae^{st} = 0$$

Thus,

$$(s^2 + 2s + 2) = (s + 1 + j)(s + 1 - j) = 0$$

which gives complex roots at $s_{1,2} = -1 \mp j$.

Hence,

$$i_{L_n} = Ae^{-(1+j)t} + Be^{-(1-j)t}$$

This means that the complete response is

$$i_L = i_{L_f} + i_{L_n} = 5 + Ae^{-(1+j)t} + Be^{-(1-j)t}$$

At $t = 0$,

$$i_L(0) = 5 + Ae^0 + Be^0 = 5 + A + B = 0$$
$$A + B = -5 \qquad \text{or} \qquad B = -A - 5$$

Also,

$$\frac{di_L(0)}{dt} = 0 - (1 + j)\,A - (1 - j)\,B = 5$$
$$-A - jA - B + jB = 5$$
$$-A - jA - (-A - 5) + j(-A - 5) = 5$$
$$-2jA + 5 - j5 = 5$$
$$A = \frac{5 - 5 + j5}{-2j} = \frac{-5}{2}$$

Then,

$$B = -A - 5 = \frac{5}{2} - 5 = \frac{-5}{2}$$

Therefore,

$$i_L = 5 + (-5/2)e^{-(1+j)t} + (-5/2)e^{-(1-j)t}$$
$$i_L(t) = 5 - 5e^{-t}\left[\frac{e^{-jt} + e^{jt}}{2}\right]$$
$$i_L(t) = \left[5 - 5e^{-t}\cos(t)\right] \text{amps } \forall\ t > 0$$

- **EVALUATE the solution and check for accuracy.**
  Check to see if the answer satisfies the initial conditions.

$$i_L(0) = 5 - 5e^{-0}\cos(0) = 5 - 5 = 0 \text{ amps}$$

This matches the initial condition that

$$i_L(0^-) = i_L(0^+) = 0 \text{ amps}$$

$$\frac{di_L(t)}{dt} = 0 + 5e^{-t}\sin(t) + 5e^{-t}\cos(t)$$

$$\frac{di_L(0)}{dt} = 5e^{-0}\sin(0) + 5e^{-0}\cos(0) = 0 + 5 = 5 \text{ amps/sec}$$

This matches the initial condition that

$$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{75}{15} = 5 \text{ amp/sec}$$

- **Has the problem been solved SATISFACTORILY? If so, present the solution; if not, then return to "ALTERNATIVE solutions" and continue through the process again.**
  This problem has been solved satisfactorily.

$$i_L(t) = \underline{\mathbf{[5 - 5e^{-t}\cos(t)]\,amps\ \forall\ t > 0}}$$

SECOND-ORDER OP AMP CIRCUITS

Problem 8.16 Given the circuit shown in Figure 8.15, find v_o in terms of v_i.

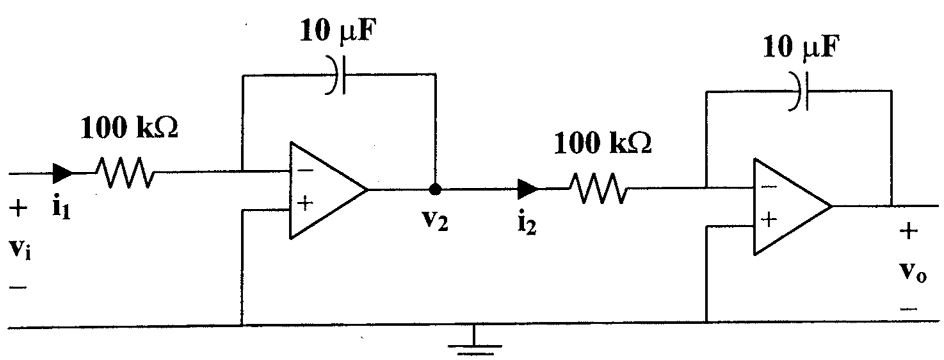


Figure 8.15

Clearly,

$$i_1 = \frac{v_i}{R_1}$$

and

$$v_2 = \frac{-1}{C_1}\int \frac{v_i}{R_1}\,dt = \frac{-1}{R_1C_1}\int v_i\,dt$$

where $R_1C_1 = (10^5)(10^{-5}) = 1$.
Hence,

$$v_2 = -\int v_i\,dt$$

Also,

$$i_2 = \frac{v_2}{R_2}$$

and

$$v_o = \frac{-1}{C_2}\int \frac{v_2}{R_2}\,dt = \frac{-1}{R_2C_2}\int v_2\,dt$$

where $R_2C_2 = (10^5)(10^{-5}) = 1$.

So,

$$v_o = -\int v_2\,dt$$

Therefore,

$$v_o = \underline{\boldsymbol{\iint v_i\,dt^2}}$$

Circuits like these have many applications, especially in analog computers.